Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 19, No. 3, (June 2020) pp. 281–293
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr
https://doi.org/10.30948/afmi.2020.19.3.281

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Reprinted from the
Annals of Fuzzy Mathematics and Informatics
Vol. 19, No. 3, June 2020

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Received 18 December 2019; Revised 17 January 2020; Accepted 28 February 2020

Abstract. In this paper, a new class of topological spaces, called maximal spaces, is introduced by means of fuzzy submaximality and fuzzy extremally disconnectedness of fuzzy topological spaces. Several characterizations of fuzzy maximal spaces are established. It is established that fuzzy maximal spaces, are fuzzy nodec spaces and fuzzy irresolvable spaces and fuzzy maximal spaces are neither fuzzy Baire spaces nor fuzzy β-Baire spaces.

2010 AMS Classification: 54 A 40, 03 E 72

Keywords: Fuzzy dense set, Fuzzy nowhere dense set, Fuzzy Baire set, Fuzzy simply* open set, Fuzzy nodec space, Fuzzy irresolvable space and fuzzy globally disconnected space.

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1. Introduction

In 1965, Zadeh[24] introduced the concept of fuzzy sets as a new approach for modeling uncertainities. The potential of fuzzy notion was realized by the researchers and has successfully been applied in all branches of Mathematics. In 1968, Chang[7] introduced the concept of fuzzy topological spaces. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concept. In the recent years, there has been a growing trend among many fuzzy topologists to introduce and study different types of fuzzy spaces in fuzzy topology.

The term "submaximal space" was first used by Bourbaki in [6]. Since its birth, the class of submaximal spaces has attracted the attention of several authors. The first systematic study of submaximal spaces was undertaken in the paper of Arhangel'skii and Collins[1]. Extremally disconnected topological spaces were introduced by Gillman and Jerison[9]. The notion of fuzzy extremally disconnected spaces was studied by Ghosh[8]. The notion of fuzzy submaximal spaces was studied by Balasubramanian[4]. In this paper, a new class of fuzzy topological spaces, called fuzzy maximal spaces, is introduced and studied. Several characterizations of fuzzy maximal spaces are established.

In this paper, a new class of topological spaces, called maximal spaces, is introduced by means of fuzzy submaximality and fuzzy extremally disconnectedness of fuzzy topological spaces. Several characterizations of fuzzy maximal spaces are established. It is established that fuzzy Baire sets in fuzzy maximal spaces are fuzzy G_δ-sets and fuzzy maximal space has no non-zero fuzzy dense and fuzzy G_δ-sets. Also it is established that fuzzy maximal spaces, are fuzzy nodec spaces, fuzzy irresolvable spaces and fuzzy maximal spaces are neither fuzzy Baire spaces nor fuzzy β-Baire spaces.

2. Preliminaries

In order to make the exposition self-contained, some basic notions and results used in the sequel are given. In this work (X, T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). Let X be a non-empty set and I, the unit interval [0,1]. A fuzzy set λ in X is a function from X into I. The null fuzzy set 0_X is the function from X into I which assumes only the value 0 and the whole fuzzy set 1_X is the function from X into I takes the value 1 only.

Definition 2.1 ([7])**.** Let (X, T) be a fuzzy topological space and λ be any fuzzy set in (X, T). The interior, the closure and the complement of λ are defined respectively as follows:

(i) $int(\lambda) = \vee\{\mu/\mu \leq \lambda, \mu \in T\}$,
(ii) $cl(\lambda) = \wedge\{\mu/\lambda \leq \mu, 1 - \mu \in T\}$,
(iii) $\lambda'(x) = 1 - \lambda(x)$, for all $x \in X$.

Lemma 2.2. [2] *For a fuzzy set λ of a fuzzy topological space X,*
(1) $1 - int(\lambda) = cl(1 - \lambda)$,
(2) $1 - cl(\lambda) = int(1 - \lambda)$.

Definition 2.3. A fuzzy set λ in a fuzzy topological space (X, T) is called a:

(i) fuzzy regular-open set, if $\lambda = intcl(\lambda)$ [2],
(ii) fuzzy regular-closed set, if $\lambda = clint(\lambda)$ [2],
(iii) fuzzy β-open set, if $\lambda \leq clintcl(\lambda)$ [5],
(iv) fuzzy β-closed set, if $\lambda \leq intclint(\lambda)$ [5].

Definition 2.4. A fuzzy set λ in a fuzzy topological space (X, T) is called a:

(i) fuzzy dense set, if there exists no fuzzy closed set μ in (X, T) such that $\lambda < \mu < 1$, i.e., $cl(\lambda) = 1$ in (X, T) [11].
(ii) fuzzy nowhere dense set, if there exists no non-zero fuzzy open set μ in (X, T) such that $\mu < cl(\lambda)$, i.e., $intcl(\lambda) = 0$ in (X, T) [11].
(iii) fuzzy first category set, if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X, T). Any other fuzzy set in (X, T) is said to be of fuzzy second category [11].
(iv) fuzzy G_δ-set in (X, T), if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where $\lambda_i \in T$ for $i \in I$ [3].
(v) fuzzy F_σ-set in (X, T), if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $1 - \lambda_i \in T$ for $i \in I$ [3].
(vi) fuzzy σ-nowhere dense set, if λ is a fuzzy F_σ-set in (X, T) such that $int(\lambda) = 0$ [14].

(vii) fuzzy simply* open set, if $\lambda = \mu \vee \delta$, where μ is a fuzzy open set and δ is a fuzzy nowhere dense set in (X, T) [18].

(viii) fuzzy somewhere dense set, if $intcl(\lambda) \neq 0$, in (X, T) [23].

(ix) fuzzy resolvable set, if for a fuzzy closed set μ in (X, T), $\{cl(\mu \wedge \lambda) \wedge cl(\mu \wedge (1 - \lambda))\}$ is a fuzzy nowhere dense in (X, T) [19].

(x) fuzzy irresolvable set, if for a fuzzy closed set μ in (X, T), $\{cl(\mu \wedge \lambda) \wedge cl(\mu \wedge (1 - \lambda))\}$ is a fuzzy somewhere dense in (X, T) [19].

(xi) fuzzy Baire set, if $\lambda = (\mu \wedge \delta)$, where μ is a fuzzy open set and δ is a fuzzy residual set in (X, T) [15].

(xii) fuzzy β-nowhere dense set, if there exists no non-zero fuzzy β-open set μ in (X, T) such that $\mu < cl(\lambda)$, i.e., β-intβ-cl$(\lambda) = 0$ in (X, T) [16].

Definition 2.5. [11] Let λ be a fuzzy first category set in a fuzzy topological space in (X, T). Then $1 - \lambda$ is called a fuzzy residual set in (X, T).

Definition 2.6. A fuzzy topological space (X, T) is called a:

(i) fuzzy Baire space, if $int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where (λ_i)'s are fuzzy nowhere dense sets in (X, T) [13].

(ii) fuzzy submaximal space, if for each fuzzy set λ in (X, T) such that $cl(\lambda) = 1$, $\lambda \in T$ [4].

(iii) fuzzy globally disconnected space, if each fuzzy semi-open set is fuzzy open in (X, T) [20].

(iv) fuzzy basically disconnected space, if the closure of each fuzzy open F_σ-set is fuzzy open in (X, T) [10].

(v) fuzzy perfectly disconnected space, if for any two non-zero fuzzy sets λ and μ defined on X with $\lambda \leq 1 - \mu$, $cl(\lambda) \leq 1 - cl(\mu)$ in (X, T) [21].

(vi) fuzzy hereditarily irresolvable space, if there is no non-zero fuzzy resolvable set in (X, T) [19].

(vii) fuzzy open hereditarily irresolvable space, if each non-zero fuzzy open set is a fuzzy irresolvable set in (X, T) [19].

(viii) fuzzy nodec space, if each non-zero fuzzy nowhere dense set is fuzzy closed set in (X, T) [13].

(ix) Fuzzy extremally disconnected space, if closure of every fuzzy open set is a fuzzy open set in (X, T) [8].

(x) fuzzy β-Baire space, if $int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where (λ_i)'s are fuzzy β-nowhere dense sets in (X, T) [16].

(xi) fuzzy resolvable space, if there exists a fuzzy dense set λ in (X, T) such that $cl(1 - \lambda) = 1$. Otherwise (X, T) is called a fuzzy irresolvable space [12].

Definition 2.7 ([17])**.** Let (X, T) be a fuzzy topological space. A fuzzy set λ defined on X is said to have the property of fuzzy Baire, if $\lambda = (\mu \wedge \delta) \vee \eta$, where μ is a fuzzy open set, δ is a fuzzy residual set and η is a fuzzy first category set in (X, T).

Theorem 2.8 ([20])**.** *If a fuzzy topological space (X, T) is a fuzzy globally disconnected space, then (X, T) is a fuzzy nodec space.*

Theorem 2.9 ([20])**.** *If a fuzzy topological space (X, T) is a fuzzy globally disconnected space, then (X, T) is a fuzzy extremally disconnected space.*

Theorem 2.10 ([20]). *If a fuzzy topological space (X, T) is a fuzzy open hereditarily irresolvable and fuzzy globally disconnected space, then (X, T) is a fuzzy submaximal space.*

Theorem 2.11 ([13]). *Let (X, T) be a fuzzy topological space. Then the following are equivalent:*
(1) *(X, T) is a fuzzy Baire space,*
(2) *$int(\lambda) = 0$, for every fuzzy first category set λ in (X, T),*
(3) *$cl(\mu) = 1$, for every fuzzy residual set μ in (X, T).*

Theorem 2.12 ([13]). *If $cl(\wedge_{i=1}^{\infty}(\lambda_i)) = 1$, where (λ_i)'s are fuzzy dense and open sets in a fuzzy topological space (X, T), then (X, T) is a fuzzy Baire space.*

Theorem 2.13 ([10]). *If a fuzzy topological space (X, T) is a fuzzy basically disconnected space in which fuzzy open sets are fuzzy F_σ-sets, then (X, T) is a fuzzy extremally disconnected space.*

Theorem 2.14 ([21]). *If a fuzzy topological space (X, T) is a fuzzy perfectly disconnected space, then (X, T) is a fuzzy extremally disconnected space.*

Theorem 2.15 ([19]). *If λ is a fuzzy irresolvable set in a fuzzy topological space (X, T), then λ and $1 - \lambda$ are fuzzy somewhere dense sets in (X, T)*

Theorem 2.16 ([19]). *If λ is a fuzzy set defined on X in a fuzzy topological space (X, T) in which fuzzy open sets are fuzzy irresolvable sets, then $1 - \lambda$ is a fuzzy somewhere dense set in (X, T).*

Theorem 2.17 ([19]). *If a fuzzy topological space (X, T) is a fuzzy hereditarily irresolvable space, then (X, T) is a fuzzy open hereditarily irresolvable space.*

Theorem 2.18 ([15]). *If λ is a fuzzy Baire set in a fuzzy topological space (X, T), then there exists a fuzzy Baire set μ in (X, T) such that $\mu \leq \lambda$.*

Theorem 2.19 ([16]). *If λ is a fuzzy σ-nowhere dense set in a fuzzy topological space (X, T) then $1 - \lambda$ is a fuzzy residual set in (X, T).*

Theorem 2.20 ([17]). *If λ is a fuzzy set with Baire property in a fuzzy topological space (X, T), then there exists a fuzzy Baire set σ in (X, T) such that $\sigma \leq \lambda$, in (X, T).*

Theorem 2.21 ([22]). *If λ is a fuzzy somewhere dense set in a fuzzy topological space (X, T), then there exist a fuzzy regular closed set η in (X, T) such that $\eta \leq cl(\lambda)$.*

Theorem 2.22 ([16]). *If λ is a fuzzy nowhere dense and fuzzy closed set in a fuzzy topological space (X, T), then λ is a fuzzy β-nowhere dense set in (X, T).*

3. Fuzzy maximal spaces

Definition 3.1. A fuzzy topological space (X, T) is called a fuzzy maximal space, if (X, T) is a fuzzy extremally disconnected and fuzzy submaximal space.

Example 3.2. Let $X = \{a, b, c\}$. Consider the fuzzy sets α, β, γ, λ, μ and δ are defined on X as follows:
$\alpha : X \to [0, 1]$ is defined as $\alpha(a) = 0.6$; $\alpha(b) = 0.5$; $\alpha(c) = 0.2$,

$\beta : X \to [0,1]$ is defined as $\beta(a) = 0.5$; $\beta(b) = 0.9$; $\beta(c) = 0.7$,
$\gamma : X \to [0,1]$ is defined as $\gamma(a) = 0.4$; $\gamma(b) = 0.5$; $\gamma(c) = 0.8$,
$\lambda : X \to [0,1]$ is defined as $\lambda(a) = 0.6$; $\lambda(b) = 0.9$; $\lambda(c) = 0.7$,
$\mu : X \to [0,1]$ is defined as $\mu(a) = 0.5$; $\mu(b) = 0.9$; $\mu(c) = 0.8$,
$\delta : X \to [0,1]$ is defined as $\delta(a) = 0.6$; $\delta(b) = 0.9$; $\delta(c) = 0.8$.

Then $T = \{0, \alpha, \beta, \gamma, \alpha\vee\beta, \alpha\vee\gamma, \beta\vee\gamma, \alpha\wedge\beta, \alpha\wedge\gamma, \beta\wedge\gamma, \alpha\vee(\beta\wedge\gamma), \gamma\vee(\alpha\wedge\beta), \beta\wedge(\alpha\vee\gamma), \alpha\vee\beta\vee\gamma, 1\}$ is a fuzzy topology on X. On computation, one can see that $cl(\alpha) = 1-\gamma = \alpha \in T$; $cl(\beta) = 1 \in T$; $cl(\gamma) = 1-\alpha = \gamma \in T$; $cl(\alpha\vee\beta) = 1 \in T$; $cl(\alpha\vee\gamma) = 1-(\alpha\wedge\gamma) = \alpha\vee\gamma \in T$; $cl(\beta\vee\gamma) = 1 \in T$; $cl(\alpha\wedge\beta) = 1-\gamma\vee(\alpha\wedge\beta) = \alpha\wedge\beta \in T$; $cl(\alpha\wedge\gamma) = 1-(\alpha\vee\gamma) = \alpha\wedge\gamma \in T$; $cl(\beta\wedge\gamma) = 1-\alpha = \gamma \in T$; $cl[\alpha\vee(\beta\wedge\gamma)] = 1-(\alpha\wedge\gamma) = \alpha\vee\gamma \in T$; $cl[\gamma\vee(\alpha\wedge\beta)] = 1-(\alpha\wedge\beta) = \gamma\vee(\alpha\wedge\beta) \in T$; $cl[\beta\wedge(\alpha\vee\gamma)] = 1-(\alpha\wedge\beta) = \gamma\vee(\alpha\wedge\beta) \in T$ and $cl(\alpha\vee\beta\vee\gamma) = 1 \in T$. Since the closure of each fuzzy open set in (X,T) is a fuzzy open set in (X,T), (X,T) is a fuzzy extremally disconnected space. Also, $cl(\lambda) = 1$, $cl(\mu) = 1$ and $cl(\delta) = 1$, implies that , λ, μ and δ are fuzzy dense sets in (X,T) and $\lambda = \alpha\vee\beta \in T$, $\mu = \beta\vee\gamma \in T$ and $\delta = \alpha\vee\beta\vee\gamma \in T$ shows that (X,T) is a fuzzy submaximal space. Thus (X,T) is a fuzzy maximal space.

Proposition 3.3. *If the fuzzy topological space (X,T) is a fuzzy maximal space, then*

(1) *for a fuzzy nowhere dense set μ in (X,T), μ is a fuzzy closed set in (X,T),*
(1) *for a fuzzy closed set δ in (X,T), $int(\delta)$ is a fuzzy closed set in (X,T),*
(1) *for a fuzzy somewhere dense set σ in (X,T), $int(\sigma) \neq 0$ in (X,T).*

Proof. (1) Let μ be a fuzzy nowhere dense set in (X,T). Then $intcl(\mu) = 0$ in (X,T). But $int(\mu) \leq intcl(\mu)$ in (X,T). This implies that $int(\mu) = 0$ in (X,T). Now $cl(1-\mu) = 1 - int(\mu) = 1 - 0 = 1$ and thus $1-\mu$ is a fuzzy dense set in (X,T). Since (X,T) is a fuzzy maximal space, (X,T) is a fuzzy submaximal space and so the fuzzy dense set $1-\mu$ is a fuzzy open set in (X,T). Hence μ is a fuzzy closed set in (X,T).

(2) Let δ be a fuzzy closed set in (X,T). Then $1-\delta$ is a fuzzy open set in (X,T). Since (X,T) is a fuzzy maximal space, by (1), $cl(1-\delta)$ is a fuzzy open set in (X,T). Thus $1-int(\delta)$ is a fuzzy open set in (X,T). So $int(\delta)$ is a fuzzy closed set in (X,T).

(3) Let σ be a fuzzy somewhere dense set in (X,T). Then $intcl(\sigma) \neq 0$, in (X,T). Suppose that $int(\sigma) = 0$ in (X,T). Then $cl(1-\sigma) = 1 - int(\sigma) = 1 - 0 = 1$. Thus $1-\sigma$ is a fuzzy dense set in (X,T). Since (X,T) is a fuzzy maximal space, (X,T) is a fuzzy submaximal space. So the fuzzy dense set $1-\sigma$ is a fuzzy open set in (X,T). Hence σ is a fuzzy closed set in (X,T) for which $cl(\sigma) = \sigma$ in (X,T). This will imply that $intcl(\sigma) = int(\sigma) = 0$, a contradiction. Therefore $int(\sigma) \neq 0$, for a fuzzy somewhere dense set σ in (X,T). □

Proposition 3.4. *If λ is a fuzzy first category set in a fuzzy maximal space (X,T), then λ is a fuzzy F_σ-set in (X,T).*

Proof. Let λ be a fuzzy first category set in (X,T). Then $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$ where (λ_i)'s are fuzzy nowhere dense sets in (X,T). Since (X,T) is a fuzzy maximal space, by Proposition 3.3 (1), (λ_i)'s are fuzzy closed sets in (X,T). Thus $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy closed sets in (X,T), implies that λ is a fuzzy F_σ-set in (X,T). □

Proposition 3.5. *If λ is a fuzzy residual set in a fuzzy maximal space (X, T), then λ is a fuzzy G_δ-set in (X, T).*

Proof. Let λ be a fuzzy residual set in (X, T). Then $1 - \lambda$ is a fuzzy first category set in (X, T). Since (X, T) is a fuzzy maximal space, by Proposition 3.4, $1 - \lambda$ is a fuzzy F_σ-set in (X, T). Thus λ is a fuzzy G_δ-set in (X, T). □

Proposition 3.6. *If λ is a fuzzy simply* open set in a fuzzy maximal space (X, T), then $\lambda = \mu \vee \delta$, where $\mu, 1 - \delta \in T$.*

Proof. Let λ be a fuzzy simply* open set in (X, T). Then $\lambda = \mu \vee \delta$, where μ is a non-zero fuzzy open set and δ is a fuzzy nowhere dense set in (X, T). Since (X, T) is a fuzzy maximal space, by Proposition 3.3 (1), the fuzzy nowhere dense set δ is a fuzzy closed set in (X, T). Thus $\lambda = \mu \vee \delta$, where $\mu, 1 - \delta \in T$. □

Proposition 3.7. *If λ is a fuzzy set defined on X in a fuzzy maximal space (X, T), then $int(\lambda) \neq 0$ in (X, T).*

Proof. Let λ be a fuzzy set defined on X in (X, T). Now $cl(\lambda)$ is a fuzzy closed set in (X, T) and then $int[cl(\lambda)]$ is a fuzzy open set in (X, T). Since (X, T) is a fuzzy maximal space, $cl(int[cl(\lambda)])$ is a non-zero fuzzy open set in (X, T). This implies that $int[cl(\lambda)] \neq 0$ and thus λ is a fuzzy somewhere dense set in (X, T). Since (X, T) is a fuzzy maximal space, by Proposition 3.3 (3), for the fuzzy somewhere dense set in (X, T), $int(\lambda) \neq 0$ in (X, T). □

Proposition 3.8. *If λ is a fuzzy first category set in a fuzzy maximal space (X, T), then λ is not a fuzzy σ-nowhere dense set in (X, T).*

Proof. Let λ be a fuzzy first category set in (X, T). Since (X, T) is a fuzzy maximal space and by Proposition 3.4, the fuzzy first category set λ is a fuzzy F_σ-set in (X, T). Then λ is a fuzzy F_σ-set in (X, T). By Proposition 3.7, $int(\lambda) \neq 0$, for the fuzzy set λ in (X, T). Thus λ is not a fuzzy σ-nowhere dense set in (X, T). □

Proposition 3.9. *If λ is a fuzzy irresolvable set in a fuzzy maximal space (X, T), then $int(\lambda) \neq 0$ and $cl(\lambda) \neq 1$ in (X, T).*

Proof. Let λ be a fuzzy irresolvable set in (X, T). Then by theorem 2.15, λ and $1 - \lambda$ are fuzzy somewhere dense sets in (X, T). Since (X, T) is a fuzzy maximal space, by Proposition 3.3 (3), $int(\lambda) \neq 0$ and $int(1 - \lambda) \neq 0$ in (X, T). By Lemma 2.2, $1 - cl(\lambda) = int(1 - \lambda) \neq 0$ and thus $1 - cl(\lambda) \neq 0$. This implies that $cl(\lambda) \neq 1$. So for a fuzzy irresolvable set λ, $int(\lambda) \neq 0$ and $cl(\lambda) \neq 1$ in (X, T). □

Proposition 3.10. *If λ is a fuzzy set defined on X in a fuzzy maximal space (X, T) in which fuzzy open sets are fuzzy irresolvable sets, then λ is not a fuzzy dense set in (X, T).*

Proof. Let λ be a fuzzy set defined on X in (X, T) in which fuzzy open sets are fuzzy irresolvable sets. Then by Theorem 2.16, $1 - \lambda$ is a fuzzy somewhere dense set in (X, T). Since (X, T) is a fuzzy maximal space, by Proposition 3.3 (3), $int(1 - \lambda) \neq 0$ in (X, T). Thus $1 - cl(\lambda) \neq 0$. So $cl(\lambda) \neq 1$. Hence λ is not a fuzzy dense set in (X, T). □

Proposition 3.11. *If λ is a fuzzy Baire set in a fuzzy maximal space (X, T), then λ is a fuzzy G_δ-set in (X, T).*

Proof. Let λ be a fuzzy Baire set in (X, T). Then $\lambda = \mu \wedge \delta$, where μ is a fuzzy open set and δ is a fuzzy residual set in (X, T). Since (X, T) is a fuzzy maximal space, by Proposition 3.5, the fuzzy residual set δ is a fuzzy G_δ-set in (X, T). Thus $\delta = \wedge_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy open sets in (X, T). Since μ and (δ_i)'s are fuzzy open sets, $(\mu \wedge \delta_i)$'s are fuzzy open sets in (X, T). Now $\lambda = \mu \wedge \delta = \mu \wedge [\wedge_{i=1}^{\infty}(\delta_i)] = \wedge_{i=1}^{\infty}(\mu \wedge \delta_i)$, where $(\mu \wedge \delta_i)$'s are fuzzy open sets in (X, T). So λ is a fuzzy G_δ-set in (X, T). □

Proposition 3.12. *If λ is a fuzzy Baire set in a fuzzy maximal space (X, T), then there exists a fuzzy G_δ-set μ in (X, T) such that $\mu \leq \lambda$.*

Proof. Let λ be a fuzzy Baire set in (X, T). Then by Theorem 2.18, for the fuzzy Baire set in (X, T), there exists a fuzzy Baire set μ in (X, T) such that $\mu \leq \lambda$. Since (X, T) is a fuzzy maximal space, by Proposition 3.11, the fuzzy Baire set μ is a fuzzy G_δ-set in (X, T). Thus, if λ is a fuzzy Baire set in (X, T), then there exists a fuzzy G_δ-set μ in (X, T) such that $\mu \leq \lambda$. □

Proposition 3.13. *If a fuzzy topological space (X, T) is a fuzzy maximal space, then (X, T) has no non-zero fuzzy σ-nowhere dense set in (X, T).*

Proof. Suppose that λ is a fuzzy F_σ-set in (X, T). Since (X, T) is a fuzzy maximal space, by Proposition 3.7, for the fuzzy set defined on X in the fuzzy maximal space (X, T), $int(\lambda) \neq 0$, in (X, T). Then λ is a fuzzy F_σ-set such that $int(\lambda) \neq 0$ in (X, T). Thus (X, T) has no non-zero fuzzy F_σ-set such that $int(\lambda) \neq 0$ in (X, T). So (X, T) has no non-zero fuzzy σ-nowhere dense set in (X, T). □

Proposition 3.14. *If λ is a fuzzy set with fuzzy Baire property in a fuzzy maximal space (X, T), then there exists a fuzzy G_δ-set μ in (X, T) such that $\mu \leq \lambda$.*

Proof. Let λ be a fuzzy set with fuzzy Baire property in (X, T). Then by Theorem 2.20, there exists a fuzzy Baire set γ in (X, T) such that $\gamma \leq \lambda$ in (X, T). Since (X, T) is a fuzzy maximal space, by Proposition 3.12, for the fuzzy Baire set γ in (X, T), there exists a fuzzy G_δ-set μ in (X, T) such that $\mu \leq \gamma$. This implies that $\mu \leq \lambda$. □

Proposition 3.15. *If λ is a fuzzy nowhere dense set in a fuzzy maximal space (X, T), then λ is a fuzzy β-nowhere dense set in (X, T).*

Proof. Let λ be a fuzzy nowhere dense set in (X, T). Since (X, T) is a fuzzy maximal space, by Proposition 3.3 (1), λ is a fuzzy closed set in (X, T). Then λ is a fuzzy nowhere dense and fuzzy closed set in (X, T). Thus by Theorem 2.22, λ is a fuzzy β-nowhere dense set in (X, T). □

4. Fuzzy maximal spaces and other fuzzy topological spaces

Proposition 4.1. *If the fuzzy topological space (X, T) is a fuzzy maximal space, then (X, T) is a fuzzy nodec space.*

Proof. Let λ be a fuzzy nowhere dense set in (X, T). Since (X, T) is a fuzzy maximal space, by Proposition 3.3 (1), λ is a fuzzy closed set in (X, T). Then the fuzzy nowhere dense set is a fuzzy closed set in (X, T). Thus (X, T) is a fuzzy nodec space. □

Remark 4.2. The converse of the above proposition need not be true. That is, a fuzzy nodec space need not be a fuzzy maximal space. For, consider the following:

Let $X = \{a, b, c\}$. The fuzzy sets α, β, γ, λ, μ and ρ are defined on X as follows:
$\alpha : X \to [0, 1]$ is defined as $\alpha(a) = 0.5$; $\alpha(b) = 0.5$; $\alpha(c) = 0.7$,
$\beta : X \to [0, 1]$ is defined as $\beta(a) = 0.6$; $\beta(b) = 0.7$; $\beta(c) = 0.5$,
$\gamma : X \to [0, 1]$ is defined as $\gamma(a) = 0.6$; $\gamma(b) = 0.5$; $\gamma(c) = 0.5$,
$\lambda : X \to [0, 1]$ is defined as $\lambda(a) = 0.4$; $\lambda(b) = 0.3$; $\lambda(c) = 0.3$,
$\mu : X \to [0, 1]$ is defined as $\mu(a) = 0.4$; $\mu(b) = 0.5$; $\mu(c) = 0.3$,
$\rho : X \to [0, 1]$ is defined as $\rho(a) = 0.6$; $\rho(b) = 0.6$; $\rho(c) = 0.6$.
Then $T = \{0, \alpha, \beta, \gamma, \alpha\vee\beta, \alpha\vee\gamma, \alpha\wedge\beta, 1\}$ is a fuzzy topology on X. On computation, one can see that the fuzzy nowhere dense sets in (X, T), $1 - \alpha, 1 - \beta, 1 - \gamma, 1 - (\alpha \vee \beta), 1 - (\alpha \vee \gamma)$, λ and μ are fuzzy closed sets in (X, T). Thus (X, T) is a fuzzy nodec space.

Since $cl(\rho) = 1$ and $\rho \notin T$, (X, T) is not a fuzzy submaximal. Eventhough (X, T) is a fuzzy extremally disconnected space, being a fuzzy non-submaximal space (X, T) is not a fuzzy maximal space.

Remark 4.3. The converse of the above proposition need not be true. For, consider the following example:

Let $X = \{a, b, c\}$. The fuzzy sets λ, μ and γ and ρ are defined on X as follows:
$\lambda : X \to [0, 1]$ is defined as $\lambda(a) = 0.8$; $\lambda(b) = 0.5$; $\lambda(c) = 0.7$,
$\mu : X \to [0, 1]$ is defined as $\mu(a) = 0.6$; $\mu(b) = 0.9$; $\mu(c) = 0.4$,
$\gamma : X \to [0, 1]$ is defined as $\gamma(a) = 0.4$; $\gamma(b) = 0.7$; $\gamma(c) = 0.8$,
$\rho : X \to [0, 1]$ is defined as $\rho(a) = 0.6$; $\rho(b) = 0.6$; $\rho(c) = 0.5$.
Then $T = \{0, \lambda, \mu, \gamma, \lambda\vee\mu, \lambda\vee\gamma, \mu\vee\gamma, \lambda\wedge\mu, \lambda\wedge\gamma, \mu\wedge\gamma, \lambda\vee(\mu\wedge\gamma), \mu\vee(\lambda\wedge\gamma), \gamma\vee(\lambda\wedge\mu), \lambda\wedge(\mu\vee\gamma), \mu\wedge(\lambda\vee\gamma), \gamma\wedge(\lambda\vee\mu), \lambda\vee\mu\vee\gamma, \lambda\wedge\mu\wedge\gamma, 1\}$ is a fuzzy topology on X. On computation, one can see that for each dense set ω $(= \lambda, \mu, \gamma, \lambda \vee \mu, \lambda \vee \gamma, \mu \vee \gamma, \lambda \wedge \mu, \lambda\wedge\gamma, \mu\wedge\gamma, \lambda\vee(\mu\wedge\gamma), \mu\vee(\lambda\wedge\gamma), \gamma\vee(\lambda\wedge\mu), \lambda\wedge(\mu\vee\gamma), \mu\wedge(\lambda\vee\gamma), \gamma\wedge(\lambda\vee\mu), \lambda\vee\mu\vee\gamma)$ and ρ, $cl(1 - \omega) \neq 1$ and thus (X, T) is a fuzzy irresolvable space. Also, $cl(rho) = 1$ and ρ is not a fuzzy open set in (X, T), implies that (X, T) is not a fuzzy submaximal space for the fuzzy open set $\lambda \wedge \mu$, $cl(\lambda \wedge \mu) = 1 - (\lambda \wedge \mu \wedge \gamma) \notin T$. So (X, T) is not a fuzzy extremally disconnected space. Hence (X, T) being a fuzzy submaximal and fuzzy extremally disconnected, (X, T) is not a fuzzy maximal space.

Proposition 4.4. *If the fuzzy topological space (X, T) is a fuzzy maximal space, then (X, T) is a fuzzy irresolvable space.*

Proof. Let λ be a non-zero fuzzy dense set in (X, T). Since (X, T) is a fuzzy maximal space, (X, T) is a fuzzy submaximal space and then λ is a fuzzy open set in (X, T). Thus $1 - \lambda$ is a fuzzy closed set in (X, T). So $cl(1 - \lambda) = 1 - \lambda \neq 1$. Hence for the fuzzy dense set λ in (X, T), $cl(1 - \lambda) \neq 1$ in (X, T). Therefore (X, T) is a fuzzy irresolvable space. □

Proposition 4.5. *If a fuzzy topological space (X, T) is a fuzzy globally disconnected and fuzzy submaximal space, then (X, T) is a fuzzy maximal space.*

Proof. Let (X, T) be a fuzzy globally disconnected space. Then by Theorem 2.9, (X, T) is a fuzzy extremally disconnected space. Thus (X, T) is a fuzzy extremally disconnected and fuzzy submaximal space. So (X, T) is a fuzzy maximal space. □

Remark 4.6. In Example 3.2, consider the fuzzy set η defined on X as follows as follows:

$$\eta(a) = 0.5; \quad \eta(b) = 0.9; \quad \eta(c) = 0.9.$$

On computation, $int(\eta) = \beta \vee \gamma$ and $clint(\eta) = 1$ and then $\eta \leq clint(\eta)$. Thus η is a fuzzy semi-open set but not a fuzzy open set in (X, T). So (X, T) is not a fuzzy globally disconnected space. Hence (X, T) is a fuzzy maximal space but not a fuzzy globally disconnected space.

Proposition 4.7. *If a fuzzy topological space (X, T) is a fuzzy open hereditarily irresolvable and fuzzy globally disconnected space, then (X, T) is a fuzzy maximal space.*

Proof. Let (X, T) be a fuzzy open hereditarily irresolvable and fuzzy globally disconnected space. Then by Theorem 2.10, (X, T) is a fuzzy submaximal space. Also, since (X, T) is a fuzzy globally disconnected space, by Theorem 2.9, (X, T) is a fuzzy extremally disconnected space. Thus (X, T) is a fuzzy extremally disconnected and fuzzy submaximal space. So (X, T) is a fuzzy maximal space. □

Proposition 4.8. *If a fuzzy topological space (X, T) is a fuzzy hereditarily irresolvable and fuzzy globally disconnected space, then (X, T) is a fuzzy maximal space.*

Proof. Let (X, T) be a fuzzy hereditarily irresolvable and fuzzy globally disconnected space. Then by Theorem 2.17, the fuzzy hereditarily irresolvable space (X, T) is a fuzzy open hereditarily irresolvable space. Thus by Proposition 4.7, (X, T) is a fuzzy maximal space. □

Proposition 4.9. *If a fuzzy topological space (X, T) is a fuzzy perfectly disconnected and fuzzy submaximal space, then (X, T) is a fuzzy maximal space.*

Proof. Let (X, T) be a fuzzy perfectly disconnected space. Then by Theorem 2.14, (X, T) is a fuzzy extremally disconnected space. Thus (X, T) is a fuzzy extremally disconnected and fuzzy submaximal space. So (X, T) is a fuzzy maximal space. □

Remark 4.10. The converse of the above proposition need not be true. That is, a fuzzy maximal space need not be a fuzzy perfectly disconnected space. For consider the following example:

In Example 3.2, consider the following fuzzy sets ρ and θ defined on X as follows:
$\rho(a) = 0.5; \quad \rho(b) = 0.5; \quad \rho(c) = 0.6,$
$\theta(a) = 0.4; \quad \theta(b) = 0.5; \quad \theta(c) = 0.4.$
On computation, one can see that $cl(\rho) = 1 - (\alpha \wedge \beta)$, $cl(\theta) = 1 - \alpha$. Clearly, $\rho \leq 1 - \theta$. But $cl(\rho) \nleq 1 - cl(\theta)$. Then (X, T) is not a fuzzy perfectly disconnected space. Thus (X, T) is a fuzzy maximal space but not a fuzzy perfectly disconnected space.

Proposition 4.11. *If a fuzzy topological space* (X, T) *is a fuzzy maximal space, then* (X, T) *is not a fuzzy Baire space.*

Proof. Let λ be a fuzzy first category set in (X, T). Then $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X, T). Since (X, T) is a fuzzy maximal space, for the fuzzy set λ in (X, T), by Proposition 3.7, $int(\lambda) \neq 0$ and thus $int(\vee_{i=1}^{\infty}(\lambda_i)) \neq 0$ in (X, T). So (X, T) is not a fuzzy Baire space. □

Proposition 4.12. *If a fuzzy topological space* (X, T) *is a fuzzy maximal space, then* (X, T) *is not a fuzzy* β*-Baire space.*

Proof. Let (λ_i)'s $(i = 1 \text{ to } \infty)$ be fuzzy nowhere dense sets in (X, T). Since (X, T) is a fuzzy maximal space, by Proposition 3.15, (λ_i)'s are fuzzy β-nowhere dense sets in (X, T). Now $int(\vee_{i=1}^{\infty}(\lambda_i)) \leq \beta\text{-}int(\vee_{i=1}^{\infty}(\lambda_i))$. Suppose that $\beta\text{-}int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$. Then $int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$ and this implies that (X, T) is a fuzzy Baire space, a contradiction to Proposition 4.11. Then $\beta\text{-}int(\vee_{i=1}^{\infty}(\lambda_i)) \neq 0$ implies that (X, T) is not a fuzzy β-Baire space, where (λ_i)'s are fuzzy β-nowhere dense sets in (X, T). □

Remark 4.13. The converse of the above proposition need not be true. That is, a fuzzy β-Baire space need not be a fuzzy maximal space. consider the following example.

Let $X = \{a, b, c\}$. and let the fuzzy sets α, θ, δ, η, λ, μ and γ be defined on X as follows:

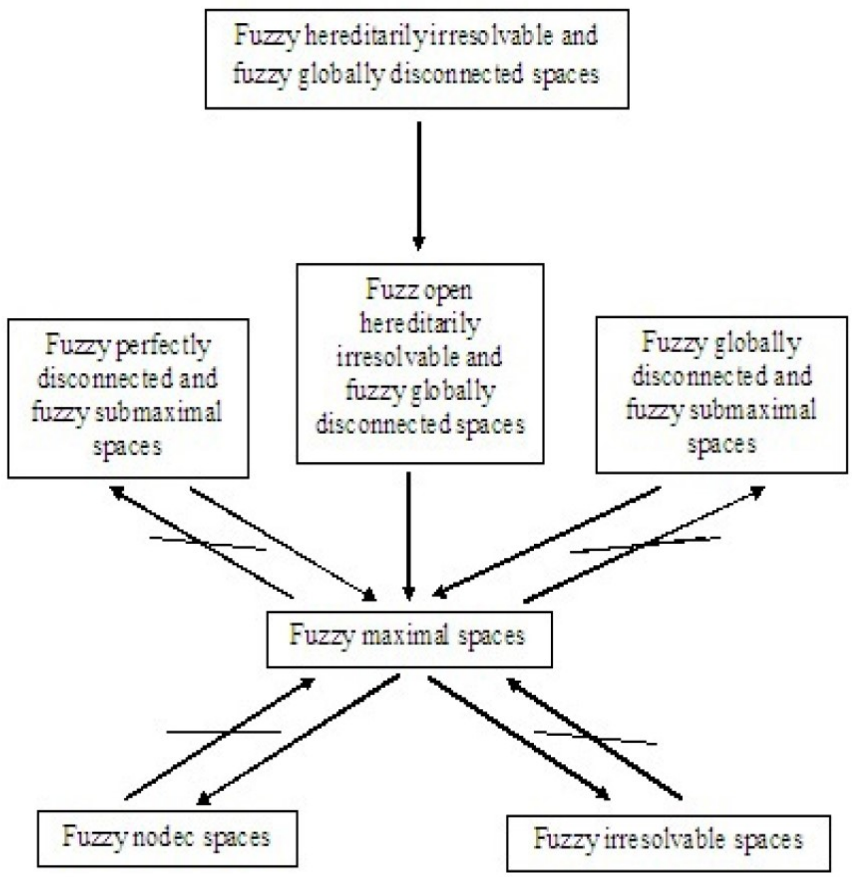
$\alpha : X \to [0, 1]$ is defined as $\alpha(a) = 0.5$; $\alpha(b) = 0.4$; $\alpha(c) = 0.6$,
$\theta : X \to [0, 1]$ is defined as $\theta(a) = 0.6$; $\theta(b) = 0.5$; $\theta(c) = 0.7$,
$\delta : X \to [0, 1]$ is defined as $\delta(a) = 0.3$; $\delta(b) = 0.1$; $\delta(c) = 0.7$,
$\eta : X \to [0, 1]$ is defined as $\eta(a) = 0.6$; $\eta(b) = 0.6$; $\eta(c) = 0.7$,
$\lambda : X \to [0, 1]$ is defined as $\lambda(a) = 0.5$; $\lambda(b) = 0.6$; $\lambda(c) = 0.6$,
$\mu : X \to [0, 1]$ is defined as $\mu(a) = 0.5$; $\mu(b) = 0.5$; $\mu(c) = 0.6$,
$\gamma : X \to [0, 1]$ is defined as $\gamma(a) = 0.4$; $\gamma(b) = 0.6$; $\gamma(c) = 0.3$.

Then $T = \{0, \alpha, \theta, 1\}$ is a fuzzy topology on X. Thus the fuzzy β-nowhere dense sets in (X, T) are γ, $1 - \alpha$, $1 - \theta$, $1 - \eta$, $1 - \lambda$ and $1 - \mu$. On computation,

$$int[\gamma \vee (1 - \alpha) \vee (1 - \theta) \vee (1 - \eta) \vee (1 - \lambda) \vee (1 - \mu)] = 0.$$

So (X, T) is a fuzzy β-Baire space. Also, $cl(\eta) = 1$ but $\eta \notin T$. Hence (X, T) is not a fuzzy submaximal. Therefore is not a fuzzy maximal space.

Remark 4.14. The inter-relations between fuzzy maximal spaces and fuzzy perfectly disconnected spaces, fuzzy hereditarily irresolvable spaces , fuzzy open hereditarily irresolvable spaces, fuzzy globally disconnected space, can be summarized as follows : fuzzy hereditarily irresolvable and fuzzy globally disconnected spaces.



Proposition 4.15. *If a fuzzy topological space (X,T) is a fuzzy maximal space, then (X,T) has no non-zero fuzzy dense and fuzzy G_δ-set in (X,T).*

Proof. Suppose that (X,T) has a non-zero fuzzy G_δ-set λ such that $cl(\lambda) = 1$ in (X,T). Then $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$ such that $cl[\wedge_{i=1}^{\infty}(\lambda_i)] = 1$ in (X,T), where (λ_i)'s are fuzzy open sets in (X,T). Now $cl[\wedge_{i=1}^{\infty}(\lambda_i)] \leq \wedge_{i=1}^{\infty}[cl((\lambda_i))]$ implies that $1 \leq \wedge_{i=1}^{\infty}[cl((\lambda_i))]$ in (X,T), i.e., $\wedge_{i=1}^{\infty}[cl((\lambda_i))] = 1$. Thus $cl((\lambda_i)) = 1$. So (λ_i)'s are fuzzy dense sets in (X,T). Hence $cl(\wedge_{i=1}^{\infty}(\lambda_i)) = 1$, where (λ_i)'s are fuzzy dense and fuzzy open sets in (X,T). By Theorem 2.12, this implies that (X,T) is a fuzzy Baire space, a contradiction to Proposition 4.9. Therefore (X,T) has no non-zero fuzzy dense and fuzzy G_δ-set in (X,T). □

Proposition 4.16. *If λ is a fuzzy Baire set in a fuzzy maximal space (X,T), then λ is a fuzzy G_δ-set in (X,T) such that $cl(\lambda) \neq 1$.*

Proof. Let λ be a fuzzy Baire set in (X,T). Then by Proposition 3.11, λ is a fuzzy G_δ-set in (X,T). Suppose that $cl(\lambda) = 1$. Then λ is a fuzzy dense and fuzzy G_δ-set in (X, T), a contradiction to Proposition 4.15. Thus the fuzzy Baire set in a fuzzy maximal space (X, T) is a fuzzy G_δ-set in (X, T) such that $cl(\lambda) \neq 1$. □

Proposition 4.17. *If fuzzy open sets are fuzzy F_σ-sets and fuzzy dense sets are fuzzy open sets in a fuzzy basically disconnected space (X, T), then (X, T) is a fuzzy maximal space.*

Proof. Let (X, T) be a fuzzy basically disconnected space in which the fuzzy open sets are the fuzzy F_σ-sets and the fuzzy dense sets are the fuzzy open sets in (X, T). Since the fuzzy open sets are the fuzzy F_σ-sets in a fuzzy basically disconnected space, by Theorem 2.13, (X, T) is a fuzzy extremally disconnected space. Also, since the fuzzy dense sets are the fuzzy open sets, (X, T) is a fuzzy submaximal space. Thus (X, T) is a fuzzy extremally disconnected and fuzzy submaximal space. So (X, T) is a fuzzy maximal space. □

Proposition 4.18. *If a fuzzy topological space (X, T) is a fuzzy open hereditarily irresolvable and fuzzy maximal space, then for any non-zero fuzzy set λ defined on X , $intcl(\lambda) \neq 0$ in (X, T).*

Proof. Let λ be any non-zero fuzzy set defined on X. Since (X, T) is a fuzzy maximal space, by Proposition 3.7, for the fuzzy set λ defined on X in the fuzzy maximal space (X, T), $int(\lambda) \neq 0$ in (X, T). Since (X, T) is a fuzzy open hereditarily irresolvable space, $int(\lambda) \neq 0$ implies that $intcl(\lambda) \neq 0$ in (X, T). □

Proposition 4.19. *If a fuzzy topological space (X, T) is a fuzzy open hereditarily irresolvable and fuzzy maximal space, then for any non-zero fuzzy set λ defined on X is a fuzzy somewhere dense set in (X, T).*

Proof. Let λ be any non-zero fuzzy set λ defined on X. Since (X, T) is a fuzzy open hereditarily irresolvable and fuzzy maximal space, by proposition 4.18, for the non-zero fuzzy set λ defined on X, $intcl(\lambda) \neq 0$ in (X, T). Then λ is a fuzzy somewhere dense set in (X, T). □

Proposition 4.20. *If a fuzzy topological space (X, T) is a fuzzy open hereditarily irresolvable and fuzzy maximal space, then any non-zero fuzzy set λ defined on X, there exists a fuzzy regular closed set η in (X, T) such that $\eta \leq cl(\lambda)$.*

Proof. Let λ be any non-zero fuzzy set defined on X. Since (X, T) is a fuzzy open hereditarily irresolvable and fuzzy maximal space, by Proposition 4.18, for the non-zero fuzzy set λ defined on X λ is a fuzzy somewhere dense set in (X, T). Then by Theorem 2.21, there exists a fuzzy regular closed set η in (X, T) such that $\eta \leq cl(\lambda)$. □

5. Conclusions

The notion of maximal spaces has been introduced in this paper by means of extremally disconnectedness with submaximality of fuzzy topological spaces. Several characterizations of fuzzy maximal spaces have been established and the inter-relations between fuzzy maximal spaces and other fuzzy topological spaces have been investigated in this paper.

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