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Abstract. In this study the notion of almost-s-Hurewicz property in ditopological texture spaces is introduced thoroughly. We study the interrelation between Hurewicz, s-Hurewicz and almost-s-Hurewicz spaces. Also we give some characterizations in terms of regular open sets and various continuous mappings. Some properties of almost-s-compactness and almost-s-stability in setting of ditopological texture spaces are discussed.

2010 AMS Classification: Primary 54A08, 54A10; Secondary 54C08, 54C10

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1. Introduction

In 1925, Hurewicz [11] (See also [12, 23]) defined a topological space X to be Hurewicz if for each sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of open covers of X there exists a sequence $(\mathcal{V}_n : n \in \mathbb{N})$ such that for each n, $\mathcal{V}_n$ is a finite subset of $\mathcal{U}_n$ and for each $x \in X$, $x \in \bigcup \mathcal{V}_n$ for all but finitely many n. Clearly every Hurewicz space is Lindelöf. As a generalization of Hurewicz spaces, authors of [24] defined a topological space X to be almost Hurewicz if for each sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of open covers of X there exists a sequence $(\mathcal{V}_n : n \in \mathbb{N})$ such that for each n, $\mathcal{V}_n$ is a finite subset of $\mathcal{U}_n$ and for each $x \in X$, $x \in \bigcup \{cl(V) : V \in \mathcal{V}_n\}$ for all but finitely many n. From the definitions above, we can easily see that the Hurewicz property implies the almost Hurewicz property. Further they showed that every regular almost Hurewicz space is Hurewicz and gave an example that there exists a Urysohn almost Hurewicz space that is not Hurewicz. On the study of Hurewicz and almost Hurewicz spaces, the readers can see the references [11, 12, 14, 22, 23, 24]. Ditopological texture spaces were introduced by Brown [3] as a natural extension of representation of lattice-valued topologies by bitopologies. The concept of ditopology is more general than general topology, bitopology and fuzzy topology in Chang's sense. An adequate introduction to the theory of texture spaces and ditopological texture spaces may be obtained from [3, 4, 5, 6, 7, 10]. We will define almost-s-Hurewicz property in the settings of ditopological texture spaces.

2. Preliminaries

We now recall some important and basic definitions used in sequel.

Texture space [3]**:** If S is a set, a texturing $\Im \subseteq P(S)$ is complete, point separating, completely distributive lattice containing S and $\varnothing$, and, for which finite join $\bigvee$ coincide with union $\bigcup$ and arbitrary meet $\bigwedge$ coincides with intersection $\bigcap$. Then the pair (S, $\Im$) is called a texture space.

A mapping $\sigma : \Im \longrightarrow \Im$ satisfying $\sigma^2(A) = A$, for each $A \in \Im$ and $A \subseteq B$ implies $\sigma(B) \subseteq \sigma(A)$, $\forall\ A,\ B \in \Im$ is called a complementation on $(S, \Im)$ and $(S, \Im, \sigma)$ is then said to be a complemented texture [3]. The sets $P_s = \bigcap\{A \in \Im \mid s \in A\}$ and $Q_s = \bigvee\{P_t \mid s \notin P_t\}$ defines conveniently most of the properties of the texture space and are known as p-sets and q-sets, respectively.

For $A \in \Im$, the core A^b of A is defined by $A^b = \{s \in S \,|\, A \not\subseteq Q_s\}$. The set A^b does not necessarily belong to $\Im$.

If $(S, P(S)), (\mathcal{L},\ \Im_2)$ are textures, then the product texture of $(S, P(S))$ and $(\mathcal{L}, \Im_2)$ is $P(S) \otimes \Im_2$ for which $\overline{P}_{(s,t)}$ and $\overline{Q}_{(s,t)}$ denotes the p-sets and q-sets, respectively. For any $s \in S,\ t \in \mathcal{L}$, we have p-sets and q-sets in the product space as following :

$$\overline{P}_{(s,t)} = \{s\} \times P_t,$$
$$\overline{Q}_{(s,t)} = (S \backslash \{s\} \times T) \cup (S \times Q_t).$$

Definition 2.1 ([5])**.** Let $(S, \Im_1), (\mathcal{L},\ \Im_2)$ be textures.

(i) $r \in \mathcal{P}(S) \otimes \Im_2$ satisfying

(R_1) $r \not\subseteq \overline{Q}_{(s,t)}$ and $P_{\acute{s}} \not\subseteq Q_s$ implies $r \not\subseteq \overline{Q}_{(\acute{s},t)}$,

(R_2) if $r \not\subseteq \overline{Q}_{(s,t)}$, then there is $\acute{s} \in S$ such that $P_s \not\subseteq Q_{\acute{s}}$ and $r \not\subseteq \overline{Q}_{(\acute{s},t)}$

is called relation.

(ii) $R \in \mathcal{P}(S) \otimes \Im_2$ such that

(CR_1) $\overline{P}_{(s,t)} \not\subseteq R$ and $P_s \not\subseteq Q_{\acute{s}}$ implies $\overline{P}_{(\acute{s},t)} \not\subseteq R$,

(CR_2) If $\overline{P}_{(s,t)} \not\subseteq R$, then there exists $\acute{s} \in S$ such that $P_{\acute{s}} \not\subseteq Q_s$ and $\overline{P}_{(\acute{s},t)} \not\subseteq R$

is called a corelation from $(S, P(S))$ to $(\mathcal{L},\ \Im_2)$.

The pair (r, R) is a direlation from $(S, \Im_1)$ to $(\mathcal{L},\ \Im_2)$.

Lemma 2.2 ([5])**.** *Let (r, R) be a direlation from $(S, \Im_1)$ to $(T, \Im_2)$, J be an index set, $A_j \in \Im_1$, $\forall j \in J$ and $B_j \in \Im_2$, $\forall j \in J$. Then*

(1) $r^{\leftarrow}(\bigcap_{j \in J} B_j) = \bigcap_{j \in J} r^{\leftarrow} B_j$ *and* $R^{\rightarrow}(\bigcap_{j \in J} A_j) = \bigcap_{j \in J} R^{\rightarrow} A_j$,

(2) $r^{\rightarrow}(\bigvee_{j \in J} A_j) = \bigvee_{j \in J} r^{\rightarrow} A_j$ *and* $R^{\leftarrow}(\bigvee_{j \in J} B_j) = \bigvee_{j \in J} R^{\leftarrow} B_j$.

Definition 2.3. Let (f, F) be a direlation from $(S, \Im_1)$ to $(\mathcal{L},\ \Im_2)$. Then (f, F) : $(S, \Im_1) \longrightarrow (\mathcal{L},\ \Im_2)$ is a difunction, if it satisfies the following two conditions:

(DF1) for $s, \acute{s} \in S, P_s \not\subseteq Q_{\acute{s}} \Longrightarrow t \in \mathcal{L}$ with $f \not\subseteq \overline{Q}_{(s,t)}$ and $\overline{P}_{(\acute{s},t)} \not\subseteq F$,

(DF2) for $t, t' \in \mathcal{L}$ and $s \in S$, $f \not\subseteq \overline{Q}_{(s,t)}$ and $\overline{P}_{(s,t')} \not\subseteq F \Longrightarrow P_{t'} \not\subseteq Q_t$.

Definition 2.4 ([5]). Let $(f, F) : (S, \Im_1) \to (\mathcal{L}, \Im_2)$ be a difunction. For $A \in \Im_1$, the image $f^{\longrightarrow}(A)$ and coimage $F^{\longrightarrow}(A)$ are defined as:

$$f^{\longrightarrow}(A) = \bigcap\{Q_t : \forall\, s,\ f \not\subseteq \overline{Q}_{(s,t)} \Longrightarrow A \subseteq Q_s\},$$

$$F^{\longrightarrow}(A) = \bigvee\{P_t : \forall\, s,\ \overline{P}_{(s,t)} \not\subseteq F \Longrightarrow P_s \subseteq A\},$$

and for $B \in \Im_2$, the inverse image $f^{\longleftarrow}(B)$ and the inverse coimage $F^{\longleftarrow}(B)$ are defined as:

$$f^{\longleftarrow}(B) = \bigvee\{P_s : \forall\, t,\ f\ \not\subseteq \overline{Q}_{(s,t)} \Longrightarrow P_t \subseteq B\},$$

$$F^{\longleftarrow}(B) = \bigcap\{Q_s : \forall\, t,\ \overline{P}_{(s,t)} \not\subseteq F \Longrightarrow B \subseteq Q_t\}.$$

For a difunction, the inverse image and the inverse co-image are equal, but the image and the co-image are usually not.

Lemma 2.5 ([5]). *For a direlation (f, F) from $(S, \Im_1)$ to $(T, \Im_2)$ the following are equivalent:*
(1) *(f, F) is a difunction,*
(2) *the following inclusion holds;*
(a) $f^{\leftarrow}(F^{\rightarrow}(A)) \subseteq A \subseteq F^{\leftarrow}(f^{\rightarrow}(A))\ \forall\ A \in \Im_1$
(b) $f^{\rightarrow}(F^{\leftarrow}(B)) \subseteq B \subseteq F^{\rightarrow}(f^{\leftarrow}(B))\ \forall\ B \in \Im_2$,
(3) $f^{\leftarrow}(B) = F^{\leftarrow}(B);\ \forall\ B \in \Im_2$.

Definition 2.6 ([5]). Let $(f, F) : (S, \Im_1) \to (\mathcal{L}, \Im_2)$ be a difunction. Then (f, F) is called surjective, if it satisfies the condition:
(SUR) for $t, t' \in \mathcal{L}$, $P_t \not\subseteq Q_{t'} \Longrightarrow \exists\ s \in S,\ f\ \not\subseteq Q_{(s,t')}$ and $\overline{P}_{(s,t)} \not\subseteq F$.
Similarly, (f, F) is called injective, if it satisfies the condition:
(INJ) For $s, \acute{s} \in S$, and $t \in \mathcal{L}$ with $f \not\subseteq \overline{Q}_{(s,t)}$ and $\overline{P}_{(\acute{s},t)} \not\subseteq F \Longrightarrow P_s \not\subseteq Q_{\acute{s}}$.

We now recall the notion of ditopology on texture spaces.

Definition 2.7 ([3]). A pair (τ, κ) of subsets of $\Im$ is said to be a ditopology on a texture space $(S, \Im)$, if $\tau \subseteq \Im$ satisfies:
(1) $S, \varnothing \in \tau$,
(2) $G_1, G_2 \in \tau$ implies $G_1 \cap G_2 \in \tau$,
(3) $G_\alpha \in \tau, \alpha \in I$ implies $\bigvee_\alpha G_\alpha \in \tau$

and $\kappa \subseteq \Im$ satisfies:
(1) $S, \varnothing \in \kappa$,
(2) $F_1, F_2 \in \kappa$ implies $F_1 \cup F_2 \in \kappa$,
(3) $F_\alpha \in \kappa, \alpha \in I$ implies $\bigcap F_\alpha \in \kappa$.

The members of τ are called open sets and members of κ are closed sets. Also τ is called topology, κ is called cotopology and (τ, κ) is called ditopology. If (τ, κ) is a ditopology on $(S, \Im)$ then $(S, \Im, \tau, \kappa)$ is called ditopological texture space.

The idea of semi-open sets in topological spaces was first introduced by Levine in 1963 in [20]. Ş. Dost extended this concept of semi-open sets from topological spaces to ditopological texture spaces in 2012 in [9]. The symbol][denotes the interior and [] denotes the closure.

It is well-known in [9] that in a ditopological texture space $(S, \Im, \tau, \kappa)$,

(1) $A \in \Im$ is semi-open if and only if there exists a set $G \in O(S)$ such that $G \subseteq A \subseteq [G]$,
(2) $B \in \Im$ is semi-closed if and only if there exists a set $F \in C(S)$ such that $]F[\subseteq B \subseteq F$,
(3) $O(S) \subseteq SO(S)$ and $C(S) \subseteq SC(S)$. The collection of all semi-open (resp. semi-closed) sets in $\Im$ is denoted by $SO(S, \Im, \tau, \kappa)$ or simply $SO(S)$ (resp. $SC(S, \Im, \tau, \kappa)$ or simply $SC(S)$). $SR(S)$ is the collection of all the semi-regular sets in S. A set A is semi-regular if A is semi-open as well as semi-closed in S,
(4) arbitrary join of semi-open sets is semi-open,
(5) arbitrary intersection of semi-closed sets is semi-closed.

If A is semi-open in ditopoloical texture space $(S, \Im, \tau, \kappa)$, then its complement may not be *semi-closed*. Every open set is semi-open, whereas a semi-open set may not be open. The intersection of two semi-open sets may not be semi-open, but intersection of an open set and a semi-open set is always semi-open.

In general, there is no connection between the semi-open and semi-closed sets, but in case of complemented ditopological texture space $(S, \Im, \sigma, \tau, \kappa)$, $A \in \Im$ is semi-open if and only if $\sigma(A)$ is semi-closed. The symbol $()_\circ$ denotes the semi-Interior and $\underline{()}$ denotes the semi-closure.

Definition 2.8 ([19]). Let $(S, \Im, \tau, \kappa)$ be a ditopological texture space and $A \in \Im$. We define

(i) the semi-closure $(\underline{A})$ of A under (τ, κ) by

$$(\underline{A}) = \bigcap\{B : B \in SC(S), \text{ and } A \subseteq B\},$$

(ii) The semi-interior $(A)_\circ$ of A under (τ, κ) by

$$(A)_\circ = \bigvee\{B : B \in SO(S), \text{ and } B \subseteq A\}.$$

Definition 2.9 ([19]). Let $(S, \Im, \tau, \kappa)$ be a ditopological texture space. A set $A \in \Im$ is said to be:

(1) semi-open, if $A \subseteq][A][$,
(2) semi-closed, if $[]A[] \subseteq A$.

Lemma 2.10. [19] *Let $(S, \Im, \sigma, \tau, \kappa)$ be a complemented ditopological texture space and $A \in \Im$. Then*

(1) $\sigma(\underline{A}) = (\sigma(A))_\circ$,
(2) $\sigma(A)_\circ = \underline{\sigma(A)}$,
(3) $(A)_\circ = \sigma(\underline{\sigma(A)})$.

For terms not defined here, the reader is referred to see [1, 2, 5, 8, 9, 15].

Definition 2.11. A difunction $(f, F) : (S, \Im_1, \tau_S, \kappa_S) \to (T, \Im_2, \tau_T, \kappa_T)$ is said to be:

(i) continuous [5], if $F^{\leftarrow}(G) \in \tau_S$, where $G \in \tau_T$,
(ii) cocontinuous [5], if $f^{\leftarrow}(K) \in \kappa_S$, where $K \in \kappa_T$,
(iii) bicontinuous [5], if it is continuous and cocontinuous.

Corollary 2.12 ([3]). *Let (f, F) be a difunction on $(S_1, \Im_1)$ to $(S_2, \Im_2)$.*

(1) *If* (f, F) *is surjective then* $F(f^{\leftarrow}(B)) = B = f(F^{\leftarrow}(B))$ *for all* $B \in \Im_2$*, in particular*

(1_a) $F(A) \subseteq f(A), \forall A \in \Im_1$,

(1_b) $\forall B_1, B_2 \in \Im_2,\ f^{\leftarrow}(B_1) \subseteq F^{\leftarrow}(B_2) \Longrightarrow B_1 \subseteq B_2$.

(2) *If* (f, F) *is surjective then* $F^{\leftarrow}(f(A)) = B = f^{\leftarrow}(F(A))$ *for all* $A \in \Im_1$*, in particular*

(2_a) $f(A) \subseteq F(A), \forall A \in \Im_1$,

(2_b) $\forall A_1, A_2 \in \Im_1, F(A_1) \subseteq f(A_2) \Longrightarrow A_1 \subseteq A_2$.

Lemma 2.13 ([10])**.** *Let* $(f, F) : (S_1, \Im_1, \tau_1, \kappa_1) \to (S_2, \Im_2, \tau_2, \kappa_2)$ *be a difunction between the ditopological texture spaces. Then the following are equivalent:*

(1) (f, F) *is cocontinuous,*

(2) $A \in \Im_1 \Longrightarrow f^{\to}[A] \subseteq [f^{\to}(A)]$,

(3) $B \in \Im_2 \Longrightarrow [f^{\leftarrow}(B)] \subseteq f^{\leftarrow}([B])$.

Lemma 2.14 ([10])**.** *Let* $(f, F) : (S_1, \Im_1, \tau_1, \kappa_1) \to (S_2, \Im_2, \tau_2, \kappa_2)$ *be a difunction between the ditopological texture spaces. Then the following are equivalent:*

(1) (f, F) *is continuous,*

(2) $A \in \Im_1 \Longrightarrow\ ]F^{\longrightarrow}(A)[\ \subseteq F^{\longrightarrow}(]A[)$,

(3) $B \in \Im_2 \Longrightarrow F^{\leftarrow}(]B[) \subseteq\ ]F^{\leftarrow}(B)[$.

Definition 2.15. Let (τ, κ) be a ditopology on a texture space $(S, \Im)$ and $A \in \Im$. The family $\{G_i : i \in I\}$ is said to be a semi-open cover of A, if $G_i \in SO(S), i \in I$ and $A \subseteq \bigvee_{i \in I} G_i$. Similarly a semi-closed cocover of A is a family $\{\mathcal{F}_i : i \in I\}$ with $\mathcal{F}_i \in SC(S)$ satisfying $\bigcap_{i \in I} \mathcal{F}_i \subseteq A$.

Definition 2.16 ([9])**.** Let $(S_i, \Im_i, \tau_i, \kappa_i), i = 1, 2$ be ditoplogical texture spaces. A difunction $(f, F) : (S_1, \Im_1) \to (S_1, \Im_2)$ is said to be:

(i) semi-continuous (resp. MS-continuous), if $f^{\leftarrow}(A) \in SO(S_1), \forall\ A \in O(S_2)$ (resp. $\forall\ A \in SO(S_2)$),

(ii) semi-cocontinuous (resp. MS-cocontinuous), if $F^{\leftarrow}(A) \in SC(S_1), \forall A \in C(S_2)$ (resp. $\forall\ A \in SC(S_2)$,

(iii) semi-bicontinuous (resp. MS-bicontinuous), if it semi-continuous and semi-cocontinuous (resp. MS-continuous and MS-cocontinuous).

Lemma 2.17 ([9])**.** *Let* $(f, F) : (S_1, \Im_1, \tau_1, \kappa_1) \to (S_2, \Im_2, \tau_2, \kappa_2)$ *be a difunction.*

(1) *The following are equivalent:*

(1_a) (f, F) *is semi-irresolute,*

(1_b) $(F^{\longrightarrow}(A))_{\circ} \subseteq F^{\longrightarrow}(A_{\circ}),\ \forall\ A \in \Im_1$,

(1_c) $f^{\leftarrow}(B_{\circ}) \subseteq (f^{\leftarrow}(B))_{\circ},\ \forall\ B \in \Im_2$.

(2) *The following are equivalent:*

(2_a) (f, F) *is semi-co-irresolute,*

(2_b) $f^{\longrightarrow}(\underline{A}) \subseteq (\underline{f^{\longrightarrow}(A)}),\ \forall\ A \in \Im_1$,

(2_c) $(\underline{F^{\leftarrow}(B)}) \subseteq F^{\leftarrow}(\underline{B}),\ \forall\ B \in \Im_2$.

Lemma 2.18 ([9])**.** *Let* $(f, F) : (S_1, \Im_1, \tau_1, \kappa_1) \to (S_2, \Im_2, \tau_2, \kappa_2)$ *be a difunction.*

(1) *The following are equivalent:*

(1_a) (f, F) *is semi-continuous,*

(1_b) $]F^{\longrightarrow}(A)[\subseteq F^{\longrightarrow}(A_\circ)$, $\forall\ A \in \Im_1$,
(1_c) $f^{\longleftarrow}(]B[) \subseteq (f^{\longleftarrow}(B))_\circ$, $\forall\ B \in \Im_2$.
(2) *The following are equivalent:*
(2_a) (f, F) *is semi-cocontinuous,*
(2_b) $f^{\longrightarrow}(\underline{A}) \subseteq [f^{\ \longrightarrow}(A)]$,$\forall\ A \in \Im_1$,
(2_c) $\underline{(F^{\longleftarrow}(B))} \subseteq F^{\longleftarrow}([B])$, $\forall\ B \in \Im_2$.

Definition 2.19. Let $(S_i, \Im_i, \tau_i, \kappa_i), i = 1, 2$ be ditoplogical texture spaces. A difunction $(f, F) : (S_1, \Im_1) \to (S_1, \Im_2)$ is said to be:
(i) s-continuous, if for each semi-open set $B \in \Im_2,\ F^{\leftarrow}(B) \in \Im_1$ is an open set,
(ii) s-cocontinuous, if for each semi-closed set $B \in \Im_2,\ f^{\ \leftarrow}(B) \in \Im_1$ is a closed set,
(iii) s-bicontinuous, if it both s-continuous and s-cocontinuous.

3. Almost-s-Hurewicz ditopological texture spaces

In 2017, Kočinac et. al. defined almost-s-Menger and almost-s-Hurewicz property in topological spaces in [21, 18]. In 2015-17, Kočinac and Özçağ [16, 17] introduced the notion of Menger and Hurewicz property in ditopological texture spaces, and Ullah and Khan in [26, 27, 28] defined semi-Hurewicz, almost-Menger, almost-s-Menger and almost-Hurewicz properties in ditopological texture spaces.

In this section, the notion of almost-s-Hurewicz and almost-co-s-Hurewicz selection properties of ditopological texture spaces are defined and studied.

Let $(S, \Im, \tau, \kappa)$ be a ditopological texture space. We use the following notation:
$\mathcal{O}$: the collection of all open covers of S,
$s\mathcal{O}$: the collection of all semi-open covers of S,
$s\mathbb{C}$: the collection of all semi-closed cocovers of S,
$\underline{s\mathcal{O}}$: the collection of all families $\mathcal{U}$ of semi-open subsets of S such that $\{\underline{U} : U \in \mathcal{U}\}$ is a cover of S,
$(s\mathbb{C})_\circ$: the collection of all families $\mathcal{F}$ of semi-closed subsets of S such that $\{(F)_\circ : F \in \mathcal{F}\}$ is a cocover of S.

Definition 3.1. Let $(S, \Im, \tau, \kappa)$ be a ditopological texture space and $\mathcal{A}$ be a member of $\Im$.

(i) $\mathcal{A}$ is said to have the almost-s-Hurewicz property for the ditopological texture space, if for each sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of semi-open covers of $\mathcal{A}$, there is a sequence $(\mathcal{V}_n : n \in \mathbb{N})$ such that for each $n \in \mathbb{N}$, $\mathcal{V}_n$ is a finite subset of $\mathcal{U}_n$ and $\mathcal{A} \subseteq \bigvee_{n\in\mathbb{N}} \bigcap_{m>n} (\underline{\bigcup \mathcal{V}_m})$, where $(\underline{\bigcup \mathcal{V}_m}) = \{(\underline{\bigcup V}) : V \in \mathcal{V}_m\}$.

We say that $(S, \Im, \tau, \kappa)$ is an almost-s-Hurewicz, if the set S is an almost-s-Hurewicz. This property is denoted by $\mathcal{U}_{fin}(s\mathcal{O}, \underline{s\mathcal{O}})$.

(ii) $\mathcal{A}$ is said to have the almost-co-s-Hurewicz property for the ditopological texture space, if for each sequence $(\mathcal{F}_n : n \in \mathbb{N})$ of semi-closed cocovers of $\mathcal{A}$, there is a sequence $(\mathcal{K}_n : n \in \mathbb{N})$ such that for each $n \in \mathbb{N}$, $\mathcal{K}_n$ is a finite subset of $\mathcal{F}_n$ and $\bigcap_{n\in\mathbb{N}} \bigvee_{m>n} (\bigcap \mathcal{K}_m)_\circ \subseteq \mathcal{A}$, where $(\bigcap \mathcal{K}_m)_\circ = \{(\bigcap K)_\circ : K \in \mathcal{K}_m\}$.

We say that $(S, \Im, \tau, \kappa)$ is an almost-co-s-Hurewicz, if the set $\varnothing$ is an almost-co-s-Hurewicz. This property is denoted by $\mathcal{U}_{cfin}(s\mathbb{C}, (s\mathbb{C})_\circ)$.

Every Hurewicz space is an almost-Hurewicz and every s-Hurewicz space is an almost-s-Hurewicz. Here we give an example of s-Hurewicz space which is an almost-s-Hurewicz in setting of ditopological texture spaces.

Example 3.2. There is a ditopological texture space which is s-Hurewicz and hence almost-s-Hurewicz.

Let $(\mathbb{R}, \Im, \tau_{\mathbb{R}}, \kappa_{\mathbb{R}})$ be the real line with the texture $\Re = \{(-\infty, r] : r \in \mathbb{R}\} \cup \{(-\infty, r) : r \in \mathbb{R}\} \cup \{\mathbb{R}, \varnothing\}$, topology $\tau_{\mathbb{R}} = \{(-\infty, r) : r \in \mathbb{R}\} \cup \{\mathbb{R}, \varnothing\}$ and cotopology $\kappa_{\mathbb{R}} = \{(-\infty, r] : r \in \mathbb{R}\} \cup \{\mathbb{R}, \varnothing\}$. Then $(\mathbb{R}, \Im, \tau_{\mathbb{R}}, \kappa_{\mathbb{R}})$ is s-Hurewicz and co-s-Hurewicz. To prove that this space is s-Hurewicz. Let $(\mathcal{U}_n : n \in \mathbb{N})$ be a sequence of semi-open covers of $\mathbb{R}$. We note that semi-open sets can be of the form $(-\infty, r)$ and $(-\infty, r]$. Write $\mathbb{R} = \cup\{(-\infty, n] : n \in \mathbb{N}\}$. For each n, $\mathcal{U}_n$ is a semi-open cover of $\mathbb{R}$, hence there is some $r_n \in \mathbb{R}$ such that $(-\infty, r_n] \subseteq (-\infty, n] \in \mathcal{U}_n$. Then the sequence $(\mathcal{V}_n : n \in \mathbb{N})$ where $\mathcal{V}_n = \{(-\infty, r_n] : n \in \mathbb{N}\}$ shows that $(\mathbb{R}, \Im, \tau_{\mathbb{R}}, \kappa_{\mathbb{R}})$ is s-Hurewicz and hence almost-s-Hurewicz.

Problem: Find an almost-s-Hurewicz ditopological texture space which is not s-Hurewicz.

Evidently we have the following diagram:

$$\begin{array}{ccc} \mathsf{Hurewicz} & \Rightarrow & \mathsf{almost-Hurewicz} \\ \Uparrow & & \Uparrow \\ \mathsf{s-Hurewicz} & \Rightarrow & \mathsf{almost-s-Hurewicz} \end{array}$$

Definition 3.3 ([25])**.** Let (τ, κ) be a ditopology on the texture space $(S, \Im)$ and $A \in \Im$.

(i) A is said to be s-compact, if whenever $\{G_\alpha : \alpha \in \nabla\}$ is a semi-open cover of A, there is a finite subset ∇_0 of ∇, with $A \subseteq \vee_{\alpha \in \nabla_0} G_\alpha$.

The ditopological texture space $(S, \Im, \tau, \kappa)$ is said to be s-compact, if S is s-compact.

Every s-compact space in the ditopologcal texture space is compact but not conversely.

(2) A is said to be co-s-compact, if whenever $\{\mathcal{F}_\alpha : \alpha \in \nabla\}$ is a semi-closed cocover of A, there is a finite subset ∇_0 of ∇, with $\cap_{\alpha \in \nabla_0} \mathcal{F}_\alpha \subseteq A$.

In particular, the ditopolgical texture space $(S, \Im, \tau, \kappa)$ is said to be co-s-compact, if $\varnothing$ is co-s-compact.

In general s-compactness and co-s-compactness are independent.

Example 3.4. There is a ditopological texture space which is s-compact and hence almost-s-Hurewicz.

Let $L = (0, 1]$ with texturing $\mathfrak{L} = \{(0, r] \,|\, r \in [0, 1]\}$ and ditopology (τ, κ) such that $\tau = \{\varnothing, L\}$ and $\kappa = \{\mathfrak{L}\}$. Then it is s-compact and hence almost-s-Hurewicz.

Example 3.5. There is a ditopological texture space which is an almost-s-Hurewicz, but not s-compact.

The above Example 3.2 is not compact (by Example 3.4 in [16]) and hence not s-compact because the open and hence semi-open cover $\mathcal{U} = \{(-\infty, n) : n \in \mathbb{N}\}$ does not contain a finite subcover. But it is s-Hurewicz (see Example 2.3 in [17]) and hence almost-s-Hurewicz.

Evidently we have the following diagram:

$$\begin{array}{ccccc} \mathsf{s-compact} & \Rightarrow & s\text{–}\mathsf{Hurewicz} & \Rightarrow & \mathsf{almost-s-Hurewicz} \\ \Downarrow & & \Downarrow & & \Downarrow \\ \mathsf{compact} & \Rightarrow & \mathsf{Hurewicz} & \Rightarrow & \mathsf{almost-Hurewicz} \end{array}$$

Definition 3.6. A ditopological texture space $(S, \Im, \tau, \kappa)$ σ is said to be -s-compact (resp. co-σ-s-compact), if there is a sequence $(A_n : n \in \mathbb{N})$ of s-compact (co-s-compact) subsets of S such that $\bigvee_{n\in\mathbb{N}} A_n = S$ (resp. $\bigcap_{n\in\mathbb{N}} A_n = \varnothing$).

Theorem 3.7. *Let $(S, \Im, \tau, \kappa)$ be a ditopological texture space.*

(1) *If $(S, \Im, \tau, \kappa)$ is σ-s-compact, then $(S, \Im, \tau, \kappa)$ has the almost-s-Hurewicz property.*

(2) *If $(S, \Im, \tau, \kappa)$ is co-σ-s-compact, then $(S, \Im, \tau, \kappa)$ has the almost-co-s-Hurewicz property.*

Proof. (1). Let $(\mathcal{U}_n : n \in \mathbb{N})$ be a sequence of semi-open covers of S. Since S is σ-s-compact, it can be represented in the form $S = \bigvee_{i\in\mathbb{N}} A_i$, where each $\mathcal{A}_i$ is s-compact $A_i \subseteq A_{i+1}$, for all $i \in \mathbb{N}$. For each $i \in \mathbb{N}$, choose a finite set $\mathcal{V}_i \subseteq \mathcal{U}_i$ such that $A_i \subseteq \bigvee \mathcal{V}_i = \bigcup \mathcal{V}_i \subseteq \underline{\bigcup \mathcal{V}_i}$. Then the sequence $(\mathcal{V}_n : n \in \mathbb{N})$ shows that S is an almost-s-Hurewicz.

(2). The proof is dual to (1). □

For complemented ditopological texture spaces, we have the following results.

Theorem 3.8. *Let $(S, \Im, \sigma)$ be a texture with the complementation σ and let $(\tau,\ \kappa)$ be a complemented ditopology on $(S, \Im, \sigma)$. Then $S \in \mathcal{U}_{fin}(s\mathcal{O},\ \underline{s\mathcal{O}})$ if and only if $\varnothing \in \mathcal{U}_{cfin}(s\mathbb{C}, (s\mathbb{C})_\circ)$.*

Proof. Let $S \in \mathcal{U}_{fin}(s\mathcal{O}, \underline{s\mathcal{O}})$ and let $(\mathcal{F}_n)_{n\in\mathbb{N}}$ be a sequence of semi-closed cocovers of $\varnothing$. Then $(\sigma(\mathcal{F}_n) = \{\sigma(F) : F \in \mathcal{F}_n\})$ and $(\sigma(\mathcal{F}_n)_{n\in\mathbb{N}})$ is a sequence of semi-open covers of S. Since $S \in \mathcal{U}_{fin}(s\mathcal{O}, \underline{s\mathcal{O}})$, there is a sequence $(\mathcal{V}_n : n \in \mathbb{N})$ of finite sets such that for each n, $\mathcal{V}_n \subseteq \sigma(\mathcal{F}_n)$ and $\bigvee_{n\in\mathbb{N}} \bigcap_{m>n} (\underline{\bigvee \mathcal{V}_m}) = S$. Thus we have that for each $n \in \mathbb{N}$, $(\sigma(\mathcal{V}_n) : n \in \mathbb{N})$ is a finite subset of $(\mathcal{F}_n)$ and

$$\varnothing = \sigma(S) = \sigma(\bigvee_{n\in\mathbb{N}} \bigcap_{m>n} (\underline{\bigvee \mathcal{V}_m})) = \bigcap_{n\in\mathbb{N}} \bigvee_{m>n} (\bigcap \sigma(\mathcal{V}_m))_\circ.$$

So $\varnothing \in \mathcal{U}_{cfin}(s\mathbb{C},\ (s\mathbb{C})_\circ)$.

Conversely, let $\varnothing \in \mathcal{U}_{cfin}(s\mathbb{C},\ (s\mathbb{C})_\circ)$ and $(\mathcal{U}_n : n \in \mathbb{N})$ be sequence of semi-open covers of S. Then $(\sigma(\mathcal{U}_n) = \{\sigma(U) : U \in \mathcal{U}_n\})$ and $(\sigma(\mathcal{U}_n)_{n\in\mathbb{N}}$ is a sequence of semi-closed cocovers of $\varnothing$. Since $\varnothing \in \mathcal{U}_{cfin}(s\mathbb{C},\ (s\mathbb{C})_\circ)$, there is a sequence $(\kappa_n)_{n\in\mathbb{N}}$ of finite sets such that for each n, $\kappa_n \subseteq \sigma(\mathcal{U}_n)$ and $\bigcap\limits_{n\in\mathbb{N}} \bigvee\limits_{m>n} (\bigcap \kappa_m)_\circ = \varnothing$. Thus we have that for each n, $\sigma(\kappa_n)$ is a finite subset $\mathcal{U}_n$, such that

$$S = \sigma(\varnothing) = \sigma(\bigcap_{n\in\mathbb{N}} \bigvee_{m>n} (\bigcap \kappa_m)_\circ) = \bigvee_{n\in N} \bigcap_{m>n} (\underline{\bigvee \sigma(\kappa_m)}).$$

So $S \in \mathcal{U}_{fin}(s\mathcal{O}, \underline{s\mathcal{O}})$. □

Theorem 3.9. *Let $(S, \Im, \sigma)$ be a texture space with complementation σ and let (τ, κ) be a complemented ditopology on $(S, \Im, \sigma)$. Then for $K \in \kappa$ with $K \neq S$, K is an almost-s-Hurewicz if and only if G is an almost-co-s-Hurewicz for some $G \in \tau$ and $G \neq \varnothing$.*

Proof. ($\Rightarrow$) Let $G \in \tau$ with $G \neq \varnothing$. Let $(\mathcal{F}_n)_{n\in\mathbb{N}}$ be a sequence of semi-closed cocovers of G. Set $K = \sigma(G)$. Then we obtain $K \in \kappa$ with $K \neq S$. Since K is an almost-s-Hurewicz, for $(\sigma(\mathcal{F}_n))_{n\in\mathbb{N}}$ the sequence of semi-open covers of K, there is a sequence $(\mathcal{V}_n : n \in \mathbb{N})$ such that for each n, $\mathcal{V}_n$ is a finite subset of $\sigma(\mathcal{F}_n)$ and $\bigvee_{n\in\mathbb{N}}\bigcap_{m>n}\underline{(\bigcup \mathcal{V}_m)}$ is a semi-open cover of K. Thus $(\sigma(\mathcal{V}_n) : n \in \mathbb{N})$ is a sequence such that for each n, $\sigma(\mathcal{V}_n)$ is a finite subset of $\mathcal{F}_n$ and $\sigma(\bigvee\limits_{n\in\mathbb{N}}\bigcap_{m>n}(\underline{\bigcup \mathcal{V}_m})) = \bigcap\limits_{n\in\mathbb{N}}\bigvee_{m>n}(\bigcap \sigma(\mathcal{V}_n))_\circ \subseteq G$ which gives G is an almost-co-s-Hurewicz.

($\Leftarrow$) $K \in \kappa$ with $K \neq S$. Let $(\mathcal{U}_n : n \in \mathbb{N})$ be a sequence of semi-open covers of K. Since $K = \sigma(G)$ but G is an almost-co-s-Hurewicz, for $(\sigma(\mathcal{U}_n) : n \in \mathbb{N})$ the sequence of semi-closed cocovers of G, there is a sequence $(\mathcal{K}_n)_{n\in\mathbb{N}}$ such that for each $n \in \mathbb{N}$, $\mathcal{K}_n$ is a finite subset of $(\sigma(\mathcal{U}_n))$ and $\bigcap_{n\in\mathbb{N}}\bigvee_{m>n}(\bigcap \mathcal{K}_m)_\circ \subseteq G$. Then $\sigma(\mathcal{K}_n)$ is a sequence such that for each $n \in \mathbb{N}$, $\mathcal{K}_n$ is a finite subset of $\mathcal{U}_n$ and $\sigma(\bigcap_{n\in\mathbb{N}}\bigvee_{m>n}(\bigcap \mathcal{K}_m)) = \bigvee\limits_{n\in\mathbb{N}}\bigcap_{m>n}\underline{(\bigcup \sigma(\mathcal{K}_m)} \supseteq K$ which gives K is an almost-s-Hurewicz. □

In 2009, Kocev in [13] proved that in the definition of almost-Menger we can use regular open sets instead of open sets.

Now we extend this idea to ditopological texture spaces and prove the following results.

Theorem 3.10. *A ditopological texture space $(S, \Im, \tau, \kappa)$ is an almost-s-Hurewicz if and only if for each sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of covers of X by* semi-regular sets*, there exists a sequence $(\mathcal{V}_n : n \in \mathbb{N})$ such that for every $n \in \mathbb{N}$, $\mathcal{V}_n$ is a finite subset of $\mathcal{U}_n$ and $\bigvee\limits_{n\in\mathbb{N}}\bigcap\limits_{m>n}(\bigcup \mathcal{V}_m)$ is a cover of S.*

Proof. Let $(\mathcal{U}_n : n \in \mathbb{N})$ be a sequence of covers of S by semi-regular open sets. Since every semi-regular set is semi-open, so $(\mathcal{U}_n : n \in \mathbb{N})$ is a sequence of semi-open covers. By assumption, there exists a sequence $(\mathcal{V}_n : n \in \mathbb{N})$ such that for every $n \in \mathbb{N}$, $\mathcal{V}_n$ is a finite subset of $\mathcal{U}_n$ and $\bigvee\limits_{n\in\mathbb{N}}\bigcap\limits_{m>n}(\bigcup \mathcal{V}_m)$ covers S.

Conversely, let $(\mathcal{U}_n : n \in \mathbb{N})$ be a sequence of semi-open covers of S. Let $(\mathcal{U}_n' : n \in \mathbb{N})$ be a sequence defined by $\mathcal{U}_n' = \{\underline{U} : U \in \mathcal{U}_n\}$. Then each $\mathcal{U}_n'$ is a cover of S by semi-regular sets. Thus there exists a sequence $(\mathcal{V}_n : n \in \mathbb{N})$ such that for every $n \in \mathbb{N}$, $\mathcal{V}_n$ is a finite subset of $\mathcal{U}_n'$ and $\bigvee\limits_{n\in\mathbb{N}}\bigcap\limits_{m>n}(\bigcup \mathcal{V}_m)$ is a cover of S. By construction, for each $n \in \mathbb{N}$ and $V \in \mathcal{V}_m$ there exists $U_V \in \mathcal{U}_n$ such that $V = \underline{U_V}$. Thus $\bigvee\limits_{n\in\mathbb{N}}\bigcap\limits_{m>n}\{\bigcup \underline{U_V} : V \in \mathcal{V}_m\}$ covers S. So $(S, \Im, \tau, \kappa)$ is an almost-s-Hurewicz space. □

As expected we have dual results for an almost-co-s-Hurewicz ditopological texture spaces, we omit the proofs.

Theorem 3.11. *A ditopological texture space $(S, \Im, \tau, \kappa)$ is almost-co-s-Hurewicz if and only if for each sequence $(\mathcal{F}_n)_{n\in\mathbb{N}}$ of cocovers of $\varnothing$ by* semi-regular sets*, there exists a sequence $(\mathcal{K}_n)_{n\in\mathbb{N}}$ such that for every $n \in N$, $\mathcal{K}_n$ is a finite subset of $\mathcal{F}_n$ and $\bigcap_{n\in\mathbb{N}} \bigvee_{m>n} (\bigcap \mathcal{K}_m)$ is a cocover of $\varnothing$.*

Lemma 3.12 ([9])**.** *Let $(S, \Im, \tau, \kappa)$ be a ditopological texture space and $\mathcal{A} \in \Im$.*

(1) $\underline{\mathcal{A}} = \mathcal{A} \cup][\mathcal{A}][$.
(2) $(\mathcal{A})_\circ = \mathcal{A} \cap [][\mathcal{A}[]$.

Theorem 3.13. *If a ditopological texture space $(S, \Im, \tau, \kappa)$ is almost-s-Hurewicz and $][\mathcal{A}][$ is finite for any $\mathcal{A} \subset S$, then S is s-Hurewicz.*

Proof. Let $(\mathcal{U}_n : n \in \mathbb{N})$ be a sequence of covers of S by semi-open sets. Since S is an almost-s-Hurewicz, there exists a sequence $(\mathcal{V}_n : n \in \mathbb{N})$ such that for every $n \in \mathbb{N}$, $\mathcal{V}_n$ is a finite subset of $\mathcal{U}_n$ and $\bigcap_{n\in\mathbb{N}} \bigvee_{m>n} \{\bigcup \underline{V} : V \in \mathcal{V}_n\}$ covers S. Since for any $\mathcal{A} \subset S$, $\underline{\mathcal{A}} = \mathcal{A} \cup][\mathcal{A}][$, by assumption, there exist finite sets $(\mathcal{F}_n : n \in \mathbb{N})$ such that $S = \bigcap_{n\in\mathbb{N}} \bigvee_{m>n} \{\bigcup V : V \in \mathcal{V}_n\} \cup \bigcap_{n\in\mathbb{N}} \bigvee_{m>n} (\bigcup \mathcal{F}_n)$. For each n let $\mathcal{W}_n$ be a set of finitely many elements of $\mathcal{U}_n$ which covers $\mathcal{F}_n$. Then the sequence $(\mathcal{V}_n \cup \mathcal{W}_n : n \in \mathbb{N})$ of finite sets witnesses that S is s-Hurewicz □

The proof of the following theorem is omitted because it is dual to the proof of the previous theorem.

Theorem 3.14. *If a ditopological texture space $(S, \Im, \tau, \kappa)$ is an almost-co-s-Hurewicz and $[]\mathcal{A}[]$ is finite for any $\mathcal{A} \subset S$, then $\varnothing$ is co-s-Hurewicz.*

Theorem 3.15. *If a ditopological texture space $(S, \Im, \tau, \kappa)$ is a semi-regular space and S is an almost-s-Hurewicz space, then S is an s-Hurewicz space.*

Proof. Let $(\mathcal{U}_n : n \in \mathbb{N})$ be a sequence of semi-open covers of S. Since S is semi-regular space, by definition, there exists for each n a semi-open cover $\mathcal{V}_n$ such that $\mathcal{V}_n^{'} = \{\underline{V} : V \in \mathcal{V}_n\}$ forms a refinement of $\mathcal{U}_n$. By assumption, there exists a sequence $(\mathcal{W}_n : n \in \mathbb{N})$ such that for each n, $\mathcal{W}_n$ is a finite subset of $\mathcal{V}_n$ and $\bigvee_{n\in\mathbb{N}} \bigcap_{m>n} \{\bigcup \mathcal{W}_m^{'} : n \in \mathbb{N}\}$ is a cover of S, where $\mathcal{W}_n^{'} = \{\underline{W} : W \in \mathcal{W}_n\}$. For every $n \in \mathbb{N}$ and every $W \in \mathcal{W}_n$, we can choose $U_W \in \mathcal{U}_n$ such that $\underline{W} \subset U_W$. Let $\mathcal{U}_n^{'} = \{U_W : W \in \mathcal{W}_n\}$. We shall prove that $\bigvee_{n\in\mathbb{N}} \bigcap_{m>n} (\bigcup \mathcal{U}_m^{'})$ is a semi-open cover of S. Let $x \in S$. Then there exists $n \in \mathbb{N}$ and $\underline{W} \in \mathcal{W}_m^{'}$ such that $x \in \underline{W}$. Thus by construction, there exists $U_W \in \mathcal{U}_n^{'}$ such that $\underline{W} \subset U_W$. So $x \in U_W$. □

Theorem 3.16. *If a ditopological texture space $(S, \Im, \tau, \kappa)$ is a semi-co-regular space and $\varnothing$ is an almost-co-s-Hurewicz space, then $\varnothing$ is a co-s-Hurewicz space.*

Proof. Let $(\mathcal{F}_n : n \in \mathbb{N})$ be a sequence of semi-closed cocovers of $\varnothing$. Since S is a semi-co-regular space, by definition, there exist for each n a semi-closed cover $\mathcal{V}_n$ such that $\mathcal{V}_n^{'} = \{(V)_\circ : V \in \mathcal{V}_n\}$ forms a refinement of $\mathcal{F}_n$. Then by assumption, there exists a sequence $(\mathcal{W}_n : n \in \mathbb{N})$ such that for each n, $\mathcal{W}_n$ is a finite subset of $\mathcal{V}_n$ and $\bigcap_{n\in\mathbb{N}} \bigvee_{m>n} (\bigcap \mathcal{W}_m^{'})$ is a cocover of $\varnothing$, where $\mathcal{W}_n^{'} = \{(W)_\circ : W \in \mathcal{W}_n\}$. For every

$n \in \mathbb{N}$ and every $W \in \mathcal{W}_m$, we can choose $F_W \in \mathcal{F}_n$ such that $F_W \subset (W)_\circ$. Let $\mathcal{F}_n^{'} = \{F_W : W \in \mathcal{W}_n\}$. We shall prove that $\bigcap_{n\in\mathbb{N}} \bigvee_{m>n} (\bigcap \mathcal{F}_m^{'})$ is a semi-closed cocover of $\varnothing$. Let $x \in S$. Then there exists $n \in \mathbb{N}$ and $(W)_\circ \in \mathcal{W}_n^{'}$ such that $x \in (W)_\circ$. Thus by construction, there exists $F_W \in \mathcal{F}_n^{'}$ such that $(W)_\circ \subset F_W$. So $x \in F_W$. □

Theorem 3.17. *Every semi-regular subset of an almost-s-Hurewicz ditoplogical texture space is an almost-s-Hurewicz.*

Proof. Let F be a semi-regular subset of almost-s-Hurewicz ditopological texture space $(S, \Im, \tau, \kappa)$ and let $(\mathcal{U}_n : n \in \mathbb{N})$ be a sequence of semi-open covers of F. Then $\mathcal{V}_n = \{\mathcal{U}_n\} \cup \{S \setminus F\}$ is a semi-open cover of S for every $n \in \mathbb{N}$. Since S is an almost-s-Hurewicz, there exist finite subsets $\mathcal{V}_n^{'}$ of $\mathcal{V}_n$ for every $n \in \mathbb{N}$. Thus $\bigvee_{n\in\mathbb{N}} \bigcap_{m>n} (\underline{V} : V \in \bigcup \mathcal{V}_m^{'}\} = S$. But F is semi-regular. So $\underline{S\backslash F} = S\backslash F$ and $\bigvee_{n\in\mathbb{N}} \bigcap_{m>n} \{\underline{\bigcup V} : V \in \mathcal{V}_m^{'}, V \neq S\backslash F\}$ covers S. □

Theorem 3.18. *Every semi-regular subset of an almost-co-s-Hurewicz ditopological texture space is an almost-co-s-Hurewicz.*

Proof. It is dual to the proof of the previous theorem. □

Theorem 3.19. *Let $(f, F) : (S_1, \Im_1, \tau_1, \kappa_1) \to (S_2, \Im_2, \tau_2, \kappa_2)$ be a semi-irresolute difunction between the ditopological texture spaces. If $\mathcal{A} \in \Im_1$ is an almost-s-Hurewicz, then $f^{\to}(\mathcal{A}) \in \Im_2$ is an almost-s-Hurewicz.*

Proof. Let $(\mathcal{U}_n : n \in \mathbb{N})$ be a sequence of τ_2-semi-open covers of $f^{\to}(\mathcal{A})$. Then by Lemma 2.5 (2) (*a*) and Lemma 2.2(2) along with semi-irresoluteness of (f, F), for each n, we have

$$\mathcal{A} \subseteq F^{\leftarrow}(f^{\to}(\mathcal{A})) \subseteq F^{\leftarrow}(\bigvee \mathcal{V}_n) = \bigvee F^{\leftarrow}(\mathcal{V}_n).$$

Thus each $F^{\leftarrow}\mathcal{V}_n$ is a τ_1-semi-open cover of $\mathcal{A}$. As $\mathcal{A}$ is an almost-s-Hurewicz, for each n, there exist finite subsets $\mathcal{V}_n \subseteq \mathcal{U}_n$ such that $\mathcal{A} \subseteq \bigvee_{n\in\mathbb{N}} \bigcap_{m>n} (\bigcup \underline{F^{\leftarrow}(\mathcal{V}_m)})$. Again by Lemma 2.5 (2) (*b*), Lemma 2.2(2) and Lemma 2.17 (2c), we have

$$f^{\to}(\mathcal{A}) \subseteq f^{\to}(\bigvee_{n\in\mathbb{N}} \bigcap_{m>n} (\bigcup \underline{F^{\leftarrow}(\mathcal{V}_m)})) \subseteq f^{\to}(\bigvee_{n\in\mathbb{N}} \bigcap_{m>n} \bigcup F^{\longleftarrow}\underline{(\mathcal{V}_m))}$$

$$\Longrightarrow f^{\to}(\bigvee_{n\in\mathbb{N}} \bigcap_{m>n} \bigcup F^{\longleftarrow}\underline{(\mathcal{V}_m))} \subseteq \bigvee_{n\in\mathbb{N}} \bigcap_{m>n} \bigcup f^{\to} F^{\longleftarrow}\underline{(\mathcal{V}_m)}$$

$$\Longrightarrow \quad f^{\to}(\mathcal{A}) \subseteq \bigvee_{n\in\mathbb{N}} \bigcap_{m>n} \bigcup \underline{\mathcal{V}_m}.$$

This proves that $f^{\to}(\mathcal{A})$ is an almost-s-Hurewicz space. □

We omit the proof of the following similar statement concerning almost-co-s-Hurewicz.

Theorem 3.20. *Let $(S_1, \Im_1, \tau_1, \kappa_1)$ and $(S_2, \Im_2, \tau_2, \kappa_2)$ be ditopological texture spaces, and let (f, F) be a semi-co-irresolute difunction between them. If $\mathcal{A} \in \Im_1$ is an almost-co-s-Hurewicz, then $F^{\to}(\mathcal{A}) \in \Im_2$ is an almost-co-s-Hurewicz.*

Theorem 3.21. *Let $(f, F) : (S_1, \Im_1, \tau_1, \kappa_1) \to (S_2, \Im_2, \tau_2, \kappa_2)$ be a semi-continuous difunction between the ditopological texture spaces. If $\mathcal{A} \in \Im_1$ is an almost-s-Hurewicz, then $f^{\to}(\mathcal{A}) \in \Im_2$ is an almost-Hurewicz.*

Proof. Let $(\mathcal{U}_n : n \in \mathbb{N})$ be a sequence of τ_2-open covers of $f^{\to}(\mathcal{A})$. Then by Lemma 2.5 (2) (*a*) and Lemma 2.2(2) along with semi-continuity of (f, F), for each n, we have

$$\mathcal{A} \subseteq F^{\leftarrow}(f^{\to}(\mathcal{A})) \subseteq F^{\leftarrow}(\bigvee \mathcal{V}_n) = \bigvee F^{\leftarrow}(\mathcal{V}_n).$$

Thus each $F^{\leftarrow}(\mathcal{V}_n)$ is a τ_1-semi-open cover of $\mathcal{A}$. As $\mathcal{A}$ is an almost-s-Hurewicz, for each n, there exist finite subsets $\mathcal{V}_n \subseteq \mathcal{U}_n$ such that $\mathcal{A} \subseteq \bigvee_{n\in\mathbb{N}} \bigcap_{m>n} (\bigcup \underline{F^{\leftarrow}(\mathcal{V}_m)})$.
Again by Lemma 2.5 (2) (*b*), Lemma 2.2(2) and Lemma 2.17 (2c) we have

$$f^{\to}(\mathcal{A}) \quad \subseteq \quad f^{\to}(\bigvee_{n\in\mathbb{N}} \bigcap_{m>n} (\bigcup \underline{F^{\leftarrow}(\mathcal{V}_m)})) \subseteq f^{\to}(\bigvee_{n\in\mathbb{N}} \bigcap_{m>n} \bigcup F^{\longleftarrow}[\mathcal{V}_m])$$

$$\Longrightarrow f^{\to}(\bigvee_{n\in\mathbb{N}} \bigcap_{m>n} \bigcup F^{\longleftarrow}[\mathcal{V}_m]) \subseteq \bigvee_{n\in\mathbb{N}} \bigcap_{m>n} \bigcup f^{\to} F^{\longleftarrow}[\mathcal{V}_m]$$

$$\Longrightarrow \quad f^{\to}\mathcal{A} \subseteq \bigvee_{n\in\mathbb{N}} \bigcap_{m>n} \bigcup [\mathcal{V}_m].$$

This proves that $f^{\to}(\mathcal{A})$ is an almost-Hurewicz space. □

Finally we note the following:

Theorem 3.22. *Let $(S_1, \Im_1, \tau_1, \kappa_1)$ and $(S_2, \Im_2, \tau_2, \kappa_2)$ be ditopological texture spaces, and let (f, F) be a semi-cocontinuous difunction between them. If $\mathcal{A} \in \Im_1$ is an almost-co-s-Hurewicz, then $F^{\to}(\mathcal{A}) \in \Im_2$ is an almost-co-Hurewicz.*

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