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Prime L-fuzzy filters of a semilattice

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Abstract. In this paper, we study the notion of prime L-fuzzy filter of a bounded distributive semilattice S with truth values in a frame L. A characterization theorem of prime (maximal) L-fuzzy filters of S is established and, the set of all prime L-fuzzy filters of S is topologized and the resulting space is discussed.

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1. Introduction

The notion of the fuzzy sets was introduced by Zadeh [5] and then, since Rosenfield [1] formulated the concept of a fuzzy subgroup of a group, several mathematicians are engaged in fuzzifying various concepts of many algebraic structures. In [4], Swamy and Swamy fuzzified some basic concepts from ring theory by introduce the notions of fuzzy ideal and fuzzy prime ideal of a ring.

In [2], the authors have introduced the notion of an L-fuzzy filter of a meet-semilattice S with truth values in a frame L and proved that the class $\mathcal{F}_L F(S)$ of L-fuzzy filters of S is an algebraic fuzzy system and, the class $\mathcal{F}_L F(S)$ is distributive if and only if S is distributive. The concept of prime ideals (filters) is vital in the study of structure theory of distributive lattices. In view of the above, in this paper, we have initiated the study of primeness and maximality in the lattice of L-fuzzy filters of a bounded distributive semilattice S.

Throughout the paper, S denote a bounded distributive semilattice and L denote a frame, that is; a complete lattice satisfying the infinite meet distributive law

$$a \wedge \Big(\bigvee_{b \in T} b \Big) = \bigvee_{b \in T} (a \wedge b),$$

for all $a \in L$ and $T \subseteq L$.

In section 2, some definitions and results to be used in the sequal are given. In section 3 and 4, we introduce the notions of prime L-fuzzy filter, and maximal L-fuzzy filter of a bounded distributive semilattice S and characterize them by proving that prime(maximal) L-fuzzy filters of S are precisely the L-fuzzy filter of the form A^P_α defined by

$$A^P_\alpha(x) = \begin{cases} 1 & \text{if } x \in P \\ \alpha & \text{if } x \notin P, \end{cases}$$

where P is a prime (maximal) filter of S and α is a meet-prime element (dual atom) in the frame L. In section 5, a topology is defined on the set of all prime L-fuzzy filters a bounded distributive semilattice S and the resulting space, denoted by X, is shown to be T_0 and T_1 spaces. Finally, we have proved that X is homeomorphic with the product space $Y \times Z$ by the mapping $(P, \alpha) \mapsto A^P_\alpha$ from $Y \times Z$ onto X, where Y is the space of prime filters of S and Z is the space of meet-prime elements in L respectively.

2. Preliminaries

In this section, we present some basic definitions, results and notations which will be needed in sequal.

Definition 2.1. A semilattice (we mean a meet-semilattice) is an algebra $S = (S, \wedge)$ with one binary operation $\wedge$ satisfying the identities

$$(x \wedge y) \wedge z = x \wedge (y \wedge z),$$
$$x \wedge y = y \wedge x,$$
$$x \wedge x = x.$$

For any $x, y, z \in S$. If we define $x \leq y$ *iff* $x \wedge y = x$ for all $x, y \in S$, then $(S, \leq)$ becomes a partially ordered set in which for any $x, y \in S$, $x \wedge y$ is the greatest lower bound of x and y in S.

Theorem 2.2. *Let $(S, \wedge)$ be a bounded semilattice and $F(S)$ be the set of all filters of S. Then $(F(S), \subseteq)$ is a complete lattice. $F(S)$ called the lattice of filters of S.*

Definition 2.3 ([2])**.** An L-fuzzy subset A of a non-empty set X is a mapping from X into a frame L. If $L = [0, 1]$, then these are the usual fuzzy subsets of X. For any $\alpha \in L$, the set $A_\alpha = \{x \in X : \alpha \leq A(x)\}$ is called the α-cut of an L-fuzzy subset A of X.

Example 2.4. For any positive integer n, $[0, 1]^n$ together with co-ordinate wise ordering is a frame, where $[0, 1]$ is the unit closed interval of real numbers. Any complete chain is a frame.

Definition 2.5 ([2])**.** An L-fuzzy subset A of a semilattice $(S, \wedge)$ is said to be an L-fuzzy filter S, if it satisfies the following conditions:

(i) $A(x_0) = 1$ for some $x_0 \in S$, (ii) $A(x \wedge y) = A(x) \wedge A(y)$ *for all* $x, y \in S$.

Theorem 2.6 ([2])**.** *The following are equivalent to each other for any L-fuzzy subset A of S:*

(1) *A is an L-fuzzy filter of S,*

(2) $A(x_0) = 1$ for some $x_0 \in S$, $A(x \wedge y) \geq A(x) \wedge A(y)$ and $x \leq y \Rightarrow A(y) \geq A(x)$ (i.e., A is an isotone) for all $x, y \in S$,

(3) A_α is a filter of S for all $\alpha \in L$.

Let X be a non-empty subset of S, and let $[X)$ denote the smallest filter containing X in S. It is well known that

$$[X) = \{x \in S : \bigwedge_{i=1}^{n} a_i \leq x \text{ for some } a_i \in X\},$$

$$[a) = \{x \in S : a \leq x\} \text{ for any } a \in S.$$

Lemma 2.7 ([2]). *Let A be an L-fuzzy filter of S and X a non-empty subset of S, and $x, y \in S$. We have*

(1) $x \in [X) \Rightarrow A(x) \geq \bigwedge_{i=1}^{m} A(a_i)$ for some $a_1, a_2, \dots a_m \in X$,

(2) $x \in [y) \Rightarrow A(x) \geq A(y)$,

(3) if S is bounded, then $A(0) < 1$ and$A(1) = 1$.

Theorem 2.8 ([2]). *$(\mathcal{F}_L F(S), \leq)$ is a complete lattice in which, for any family $\{A_i : i \in \Delta\}$ of fuzzy filters of S, the g.l.b and l.u.b are given by*

$$\bigwedge_{i \in \Delta} A_i = \text{ The point-wise infimum of } A_i' s,$$

$$\bigvee_{i \in \Delta} A_i = \text{ The point-wise infimum of } \{A \in \mathcal{F}_L F(S) : A_i \leq A \text{ for all } i \in \Delta\}.$$

Theorem 2.9 ([2]). *Let A be an L-fuzzy subset of S. Then the fuzzy filter $\bar{A}$ generated by A is given by*

$$\bar{A}(x_0) = 1 \quad \text{for some } x_0 \in S,$$

$$\bar{A}(x) = \bigvee \Big\{ \bigwedge_{i=1}^{n} A(a_i) : a_1, a_2, \dots a_n \in S, \quad \bigwedge_{i=1}^{n} a_i \leq x \Big\}$$

for any $x_0 \neq x \in S$.

Recall that a semilattice $(S, \wedge)$ is said to be distributive if for any $a, b, c \in S$, $b \wedge c \leq a$ implies there exists $b_1, c_1 \in S$ such that $b_1 \geq b$, $c_1 \geq c$ and $a = b_1 \wedge c_1$.

Theorem 2.10 ([2]). *Let $(S, \wedge)$ be a bounded semilattice. Then the following are equivalent to each other:*

(1) $\mathcal{F}_L F(S)$ is a distributive lattice,

(2) $F(S)$ is a distributive lattice,

(3) S is distributive.

3. Prime L-fuzzy filters

Recall that a proper filter P of S is said to be prime, if for any filters F and G of S, $F \cap G \subseteq P$ implies that either $F \subseteq P$ or $G \subseteq P$ (equivalently, if for any $x, y \in S$, $x \notin P$ and $y \notin P$ imply the existance of $z \in S$ such that $x \leq z$, $y \leq z$ *and* $z \notin P$). The following definition is analogous to that of a prime filter of S.

Definition 3.1. A non-constant L-fuzzy filter A of S is said to be prime, if for any L-fuzzy filters B and C of S,

$$B \wedge C \leq A \Rightarrow B \leq A \textit{ or } C \leq A.$$

In other words, A is meet-prime element in the lattice $\mathcal{F}_L F(S)$ of L-fuzzy filters of S.

For any $\alpha \in L$, the interval $[\alpha, 1]$ is a frame under the induced operations on L and for any filter F of S, the L-fuzzy subset A_α^F of S can be considered as the characteristic map $\chi_F : S \to [\alpha, 1]$ and hence A_α^F is an $[\alpha, 1]$-fuzzy filter of S and hence an L-fuzzy filter of S. Also for any filters F and G of S and $\alpha, \beta \in L$, it can be easily proved that

$$A_\alpha^F \leq A_\beta^G \Leftrightarrow F \subseteq G \textit{ and } \alpha \leq \beta,$$

$$A_\alpha^{F \cap G} = A_\alpha^F \wedge A_\alpha^G.$$

Theorem 3.2. *Let P be a filter of S and $\alpha \in L$. Then A_α^P is a prime L-fuzzy filter of S iff P is a prime filter of S and α is a meet-prime element in L.*

Proof. Suppose A_α^P is prime. Then, since A_α^P is non-constant, $A_\alpha^P(x) \neq 1$ for some $x \in S$, $x \notin P$ *and* $\alpha \neq 1$. Thus P is proper.

Let F and G be two filters of S such that $F \cap G \subseteq P$. Then $A_\alpha^{F \cap G} \subseteq A_\alpha^P$. Since A_α^P is prime, $A_\alpha^F \leq A_\alpha^P$ or $A_\alpha^G \leq A_\alpha^P$ which implies that $F \subseteq P$ or $G \subseteq P$. Thus P is prime.

Let $\beta, \gamma \in L$ *and* $\beta \wedge \gamma \leq \alpha$. Then $A_\beta^P \wedge A_\gamma^P = A_{\beta\wedge\gamma}^P \leq A_\alpha^P$. Again since A_α^P is prime, we get $\beta \leq \alpha$ *or* $\gamma \leq \alpha$. Thus α is meet-prime element in L.

Conversely, suppose P is a prime filter of S and α is meet-prime element in L. Let $A, B \in \mathcal{F}_L F(S)$ such that $A \nleq A_\alpha^P$ and $B \nleq A_\alpha^P$. Then $A(x) \nleq A_\alpha^P(x)$ and $B(y) \nleq A_\alpha^P(y)$ for some $x, y \in S$, which implies $A_\alpha^P(x) = \alpha = A_\alpha^P(y)$. Thus $x \notin P$ and $y \notin P$. Since P is prime, there exists $z \in S$ such that $x \leq z$ and $y \leq z$ and $z \notin P$. Now $A(x) \nleq \alpha$ and $B(y) \nleq \alpha$ and thus $A(x) \wedge B(y) \nleq \alpha$, since α is meet prime. Since A and B are isotones, $A(x) \wedge B(y) \leq A(z) \wedge B(z) = (A \wedge B)(z)$. So $(A \wedge B)(z) \nleq A_\alpha^P(z)$. Hence $A \wedge B \nleq A_\alpha^P$. Therefore A_α^P is prime. □

Theorem 3.3. *Let A be an L-fuzzy fileter of S. Then A is prime iff the following are satisfied:*

(1) $\mid Im(A) \mid= 2$*; that is A is two-valued,*
(2) *for any $x \in S$, either $A(x) = 1$ or $A(x)$ is meet-prime element in L,*
(3) A_1*; the* 1*-cut of A is prime filter of S.*

Proof. Suppose A is prime. To prove (1), we observe that A assume at least two values, for otherwise A is constant. Since $A(1) = 1$, 1 is necessarily a value of A. Let $\alpha, \beta \in L - \{1\}$ and $A(a) = \alpha, A(b) = \beta$ for some $a, b \in S$. Let $F = \{x \in S : A(x) = 1\}$. Then $F = A_1$ and thus F is a filter of S. Define L-fuzzy subsets B and C of S as follows:

$$B(x) = \begin{cases} 1 & \text{if } x \in [a) \\ 0 & \text{if } x \notin [a) \end{cases} \quad \text{and} \quad C(x) = \begin{cases} 1 & \text{if } x \in F \\ \alpha & \text{if } x \notin F. \end{cases}$$

Then clearly $B = \chi_{[a)}$, where $[a)$ is the filter generated by a in S and $C = A_\alpha^F$. Hence

B and C are L-fuzzy filter of S. Since A_α is a filter of S and $a \in A_\alpha$, $[a) \subseteq A_\alpha$ and $\alpha = A(a) \leq A(x)$ for all $x \in [a)$, it follows that $B \wedge C \leq A$. Since A is prime, $B \leq A$ or $C \leq A$, but $B \nleq A$ by $B(a) = 1$ and $A(a) = \alpha < 1$. So $C \leq A$, In particular, $\alpha = C(b) \leq A(b) = \beta$ so that $\alpha \leq \beta$. Similarly, $\beta \leq \alpha$. Therefore $\alpha = \beta$. Hence $\mid Im(A) \mid = 2$.

To prove (2), let $x \in S$ and $A(x) < 1$. Let $\alpha, \beta \in L$ such that $\alpha \wedge \beta \leq A(x)$. Consider the L-fuzzy filters A_α^F and A_β^F of S. By (1), $A(y) = A(x)$ for all $y \in S - F$. Now $A_\alpha^F \wedge A_\beta^F \leq A$. Since A is prime, $A_\alpha^F \leq A$ or $A_\beta^F \leq A$ which implies that $\alpha \leq A(x)$ or $\beta \leq A(x)$, since $x \notin F$. Hence $A(x)$ is meet-prime element in L.

To prove (3), by (1), F is a proper filter of S. Let G and H be two filters of S such that $G \cap H \subseteq F$. Since A is prime and $\chi_G \wedge \chi_H \leq A$ which implies $\chi_G \leq A$ or $\chi_H \leq A$. Then $G \subseteq F$ or $H \subseteq F$. Thus F is prime filter of S.

Conversely, suppose the conditions (1), (2) and (3) are satisfied. Since $A(1) = 1$, let α be a value of A other than 1. Then α is a meet-prime element in L. Let $F = \{x \in S : A(x) = 1\}$. Then by (3), F is a prime filter of S and for any $x \in S$, we have

$$A(x) = \begin{cases} 1 & \text{if } x \in F \\ \alpha & \text{if } x \notin F. \end{cases}$$

Thus $A = A_\alpha^F$ which is prime L-fuzzy filter of S by Theorem 3.2. □

Theorems 3.2 and 3.3 yield the following.

Theorem 3.4. *Let A be an L-fuzzy filter of S. Then A is a prime L-fuzzy filter of S iff there exists a prime filter P of S and a meet-prime element α in L such that $A = A_\alpha^P$.*

Corollary 3.5. *Let F be any filter of S. Then χ_F is a prime L-fuzzy filter of S iff F is a prime filter of S and 0 is a meet-prime element in L.*

Corollary 3.6. *The mapping $(P, \alpha) \mapsto A_\alpha^P$ establishes a one-to-one correspondence between the pairs (P, α), where P is a prime filter of S and α is a meet-prime element in L, and the prime L-fuzzy filters of S.*

Finally in this section we characterize minimal prime L-fuzzy filters of S. Let us recall that a prime filter P of S containing a filter F is said to be a minimal prime filter containing F, if there is no prime filter of S containing F and properly contained in P. A minimal prime filter belonging to the filter $[1)$ is simply called a minimal prime filter of S. As usual, a minimal member in the set of all prime L-fuzzy filters of S under the point-wise ordering is called a minimal prime L-fuzzy filter of S, equivalently, a minimal prime L-fuzzy filter belonging to the L-fuzzy filter $\chi_{[1)}$ is a minimal prime L-fuzzy filter.

The following are simple and straightforward verification.

Theorem 3.7. *Let A be an L-fuzzy filter of S. Then A is a minimal prime iff $A = A_\alpha^P$ for some minimal prime filter P of S and a minimal meet-prime element α in L.*

Theorem 3.8. *Suppose that 0 is a meet-prime element in L. Then an L-fuzzy filter A of S is a minimal prime iff $A = \chi_P$ for some minimal prime filter P of S. Also,*

$P \mapsto \chi_P$ is a bijection of the set of minimal prime filters of S onto the set of minimal prime L-fuzzy filters of S.

4. Maximal L-fuzzy filters

In this section, we introduce the notion of a maximal L-fuzzy filter of a bounded distributive semilattice S as a maximal element in the set of all non-constant L-fuzzy filters of S under the point-wise ordering and characterize them in terms of maximal filters of S and dual atoms in the frame L. If L is the unit interval $[0, 1]$ of real numbers, then, since L has no dual atom, S can not have any maximal L-fuzzy filter, even though S may possess plenty of maximal filters.

For any L-fuzzy subset A of S and $\alpha \in L$, we define an L-fuzzy subset $A \vee \alpha$ of S by

$$(A \vee \alpha)(x) = A(x) \vee \alpha \ for\ all\ x \in S.$$

Then A is an L-fuzzy filter of S *iff* $A \vee \alpha$ is an L-fuzzy filter of S for all $\alpha \in L$.

Theorem 4.1. *Let A be an L-fuzzy filter of S. Then A is maximal iff $A = A_\alpha^M$ for some maximal filter M of S and a dual atom α in L.*

Proof. Suppose A is maximal and let $M = \{x \in S : \ A(x) = 1\}$. Then M is a proper filter of S. Since $A(1) = 1$, we shall prove that A assumes exactly one value other than 1. Since M is proper, $A(x) < 1$ for some $x \in S$. Let a, $b \in S$ such that $A(a) < 1$ and $A(b) < 1$. Put $A(a) = \alpha$ and $A(b) = \beta$. Now $A \vee \alpha$ and $A \vee \beta$ are L-fuzzy filters of S. Also, $(A \vee \alpha)(a) = A(a) \vee \alpha = \alpha \vee \alpha = \alpha < 1$ and $(A \vee \beta)(b) = A(b) \vee \beta = \beta \vee \beta = \beta < 1$ so that $A \leq A \vee \alpha < 1$ and $A \leq A \vee \beta < 1$. By the maximality of A, $A = A \vee \alpha = A \vee \beta$ which implies $\alpha = \beta$. Thus A assumes exactly one value, say α other than 1 so that

$$A(x) = \begin{cases} 1 & \text{if } x \in M \\ \alpha & \text{if } x \notin M. \end{cases}$$

So $A = A_\alpha^M$. Let N be a proper filter of S such that $M \subseteq N$. Then $A = A_\alpha^M \leq A_\alpha^N \neq 1$. By the maximality of A, $A_\alpha^M = A_\alpha^N$ which implies $M = N$. Thus M is a maximal filter of S. Also, if $\alpha \leq \beta < 1$ in L, then $A = A_\alpha^M \leq A_\beta^N \neq 1$. Again by the maximality of A, $A_\alpha^M = A_\beta^N$ and hence $\alpha = \beta$. Therefore α is a dual atom in L.

Conversely, suppose that M is a maximal filter of S and α is a dual atom in L such that $A = A_\alpha^M$. Since M is a proper, there exist $b \in S$ such that $b \notin M$. Then $A(b) = A_\alpha^M(b) = \alpha < 1$. Thus A is non-constant. Let B be an L-fuzzy filter of S such that $A \leq B < 1$. Then $M \subseteq \{x \in S : B(x) = 1\} \subsetneq S$. Since M is maximal, $M = \{x \in S : B(x) = 1\}$ that is $B(x) = 1$ for all $x \in M$, and for any $x \notin M$, $\alpha = A_\alpha^M(x) \leq B(x) < 1$. Thus $\alpha = B(x)$ which implies that $A = B$. So A is maximal L-fuzzy filter of S. □

Corollary 4.2. *The mapping $A_\alpha^M \mapsto (M, \alpha)$ is a one-to-one correspondence between maximal L-fuzzy filters of S and the pairs (M, α) where M is a maximal filter of S and α is a dual atom in L.*

5. Topology on the set X

In this section, we introduce the hull-kernel topology on the set of all prime L-fuzzy filters of S. Let us recall from Theorem 3.4 that, every prime L-fuzzy filter of S is of the form A_β^P for some prime filter P of S and meet-prime element β in L.

Definition 5.1. Let X be the set of all prime L-fuzzy filters of S. For any $x \in S$ and $\alpha \in L$, define

$$X(x, \alpha) = \{A_\beta^P \in X : x \notin P \text{ and } \alpha \nleq \beta\},$$
$$X(A) = \{A_\beta^P \in X : A \nleq A_\beta^P\},$$

for any L-fuzzy subset A of S.

Theorem 5.2. (1) *For any $x \in S$ and $\alpha \in L$, there exists an L-fuzzy filter A of S such that $X(x, \alpha) = X(A)$.*
(2) *For any L-fuzzy filter A of S, $X(A) = \bigcup\limits_{x \in S} X(x, A(x))$*

Proof. (1) Let $x \in S$ and $\alpha \in L$. If $\alpha = 0$, then

$$X(x, \alpha) = X(x, 0) = \varnothing = X(\chi_{[1)}).$$

If $\alpha \neq 0$, then define an L-fuzzy subset A of S by

$$A(y) = \begin{cases} 1 & \text{if } y = 1 \\ \alpha & \text{if } 1 \neq y \in [x) \\ 0 & \text{if } y \notin [x). \end{cases}$$

Then clearly A is an L-fuzzy filter of S. Let $A_\beta^P \in X(x, \alpha)$. Then

$$A(x) = \alpha \nleq \beta = A_\beta^P(x).$$

Thus $A \nleq A_\beta^P$. So $A_\beta^P \in X(A)$.

On the other hand, if $A \nleq A_\beta^P$, then there exists $1 \neq y \in S$ such that $A(y) \nleq A_\beta^P(y)$. Thus $A(y) \neq 0$ and $A_\beta^P(y) \neq 1$. So $y \in [x)$, $y \notin P$, so that $A_\beta^P(y) = \beta$ which implies that $x \notin P$ and $\alpha \nleq \beta$. Hence $A_\beta^P \in X(x, \alpha)$. Therefore $X(x, \alpha) = X(A)$.

(2) Let A be an L-fuzzy filter of S and $A_\alpha^P \in X(A)$. Then $A \nleq A_\alpha^P$. Thus $A(x) \nleq A_\alpha^P(x)$ for some $x \in S$, so that $A(x) \neq 0$ and $A_\alpha^P(x) \neq 1$, $A(x) \nleq \alpha$. So $A_\alpha^P \in X(x, A(x))$.

On the other hand, suppose $A_\alpha^P \in X(x, A(x))$ for some $x \in S$. Then $x \notin P$ and $A(x) \nleq \alpha$. Thus $A(x) \nleq A_\alpha^P(x)$, $A \nleq A_\alpha^P$. So $A_\alpha^P \in X(A)$. Hence $X(A) = \bigcup\limits_{x \in S} X(x, A(x))$. □

By above Theorem, the class $\{X(x, \alpha) : x \in S \text{ and } \alpha \in L\}$ forms a base for a topology on X. This topology on X is called the hull-kernal topology or the Stone topology, in honour of Stone [3]. Now for any L-fuzzy filter A of S, we define the hull of A by

$$h(A) = \{A_\alpha^P \in X : A \leq A_\alpha^P\}$$

Then $h(A)$ closed in X, since $X - h(A) = X(A)$ which is open. Also it can be easily seen that for any $A \in X$, $\overline{\{A\}} = h(A)$, where $\overline{\{A\}}$ denote the closure of $\{A\}$.

Theorem 5.3. *The space X is T_0.*

Proof. Let $A, B \in X$ and let $\overline{\{A\}} = \overline{\{B\}}$. Then $h(A) = h(B)$ which implies that $A = B$. Thus the map $A \mapsto \overline{\{A\}}$ is one-one, so that X is a T_0-space. □

Theorem 5.4. *The space X is T_1 iff every prime filter of S is maximal and every meet-prime element in L is a dual atom.*

Proof. It follows from the fact that for any $A \in X$, $\overline{\{A\}} = h(A)$ □

Let Y be the set of all prime filters of S and Z the set of all meet-prime elements in the frame L. Consider the hull-kernal topologies on Y and Z. Then the class $\{Y(x) : x \in S\}$, where $Y(x) = \{P \in Y : x \notin P\}$ is a base for a topolgy on Y. Simillarly the class $\{Z(\alpha) : \alpha \in L\}$, where $Z(\alpha) = \{\beta \in L : \alpha \nleq \beta\}$ is a base for a topology on Z.

Theorem 5.5. *The space X is homeomorphic with the product space $Y \times Z$.*

Proof. Define $f : Y \times Z \to X$ by

$$f(P, \alpha) = A^P_\alpha$$

Then by Theorem 3.4, f is a bijection. Let $X(x, \alpha)$ be a basic open set in X. Then

$$\begin{aligned} f^{-1}(X(x,\alpha)) &= \{(P,\beta) \in Y \times Z : \ A^P_\beta \in X(x,\alpha)\} \\ &= \{(P,\beta) \in Y \times Z : \ x \notin P \text{ and } \alpha \nleq \beta\} \\ &= Y(x) \times Z(\alpha) \end{aligned}$$

which is a basic open set in the product space $Y \times Z$. Hence f is continuous. Also, $f(Y(x) \times Z(\alpha)) = X(x, \alpha)$ so that f is an open mapping. Thus f is a homeomorphism. □

6. Conclusions

In this work, we have introduced the concept of prime L-fuzzy filters and maximal L-fuzzy filters of a bounded distributive semilattice S with truth values in a frame L and we have furnished characterizations of prime L-fuzzy filters and maximal L-fuzzy filters. Afterwards, we have also considered a topology on the set of all prime L-fuzzy filters and studied the resulting space. In our future work, we will consider L-fuzzy prime(maximal) filters in semilattices which are weaker than that of prime(maximal) L-fuzzy filters.

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