Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 20, No. 3, (December 2020) pp. 235–242
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr
https://doi.org/10.30948/afmi.2020.20.3.235

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Reprinted from the
Annals of Fuzzy Mathematics and Informatics
Vol. 20, No. 3, December 2020

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Received 11 June 2020; Revised 23 July 2020; Accepted 29 July 2020

Abstract. The aim of this paper is to introduce the concept of ideals and prime (semiprime) ideals of the nearness hemiring theory defined in weak nearness approximation spaces and to introduce some properties of such ideals.

2010 AMS Classification: 03E72, 03E75, 03E99, 05C38, 15A15, 16Y60, 16Y99, 16W99

Keywords: Rough sets, Near sets, Semigroups, Semirings, Nearness approximation spaces,Weak nearness approximation spaces, Nearness semigroups, Nearness rings, Nearness semirings, Nearness hemiring.

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1. Introduction

Peters studied near set theory that is a generalization of rough sets [17, 18] in 2002. Peters defined an indiscernibility relation by using the features of the objects to find the nearness of objects [21]. After that, he generalized approach theory of the nearness of non-empty sets resembling each other [19, 20, 23].

In 2012, İnan and Öztürk studied the concept of nearness groups [3, 4] (and other algebraic approaches of near sets in [1, 5, 8, 9, 10, 11, 12, 14, 15, 16]).

Recently, Öztürk considered nearness semirings and also analyzed some properties of nearness semirings and ideals [7]. Also, Öztürk et al. introduced some properties of prime ideals in nearness semirings and analyzed their basic properties [13].

In this paper, we are to introduce the concept of the nearness hemiring theory and also to analyze some properties of ideals and prime (semiprime) ideals.

2. Preliminaries

An object description is determined by means of a tuple of function values $\Phi(x)$ associated with an object $x \in X$, which is a subset of an object space $\mathcal{O}$. Assume that $B \subseteq \mathcal{F}$ is a given set of functions representing features of sample objects $X \subseteq \mathcal{O}$. Let $\varphi_i \in B$, where $\varphi_i : \mathcal{O} \to \mathbb{R}$ (set of reals). In combination, the functions representing object features provide a basis for an object description $\Phi : \mathcal{O} \to \mathbb{R}^L, \Phi(x) = (\varphi_1(x), \varphi_2(x), ..., \varphi_L(x))$ a vector containing measurements (returned values) associated with each functional value $\varphi_i(x)$, where the description length $\mid \Phi \mid = L$ (See [19]).

The important thing to notice is the choice of functions $\varphi_i \in B$ used to describe an object of interest. Sample objects $X \subseteq \mathcal{O}$ are near each if and only if the objects have similar descriptions. Recall that each φ defines a description of an object. Then let $\triangle_{\varphi_i}$ denote $\triangle_{\varphi_i} = \mid \varphi_i(\acute{x}) - \varphi_i(x) \mid$, where $\acute{x}, x \in \mathcal{O}$. The difference φ leads to a description of the indiscernibility relation " $\sim_B$ "introduced by Peters in [19].

Definition 2.1 ([19])**.** Let $x, \acute{x} \in \mathcal{O}, B \subseteq \mathcal{F}$.

$$\sim_B = \{(x, \acute{x}) \in \mathcal{O} \times \mathcal{O} \mid \triangle_{\varphi_i} = 0 \text{ for all } \varphi_i \in B\}$$

is called the indiscernibility relation on $\mathcal{O}$, where description length $i \leq \mid \Phi \mid$.

Comparing object descriptions is the basic idea in the near set approach to object recognition. Sets of object $X, \acute{X}$ are called near each other if those sets contain the objects with at least partial matching descriptions.

Definition 2.2 ([19])**.** Let $X, \acute{X} \subseteq \mathcal{O}, B \subseteq \mathcal{F}$. Then X is called near $\acute{X}$, if there exists $x \in X, \acute{x} \in \acute{X}, \varphi_i \in B$ such that $x \sim_{\varphi_i} \acute{x}$.

A weak nearness approximation space is a tuple $(\mathcal{O}, \mathcal{F}, \sim_{B_r}, N_r(B))$, where the approximation space is defined with respect to a set of perceived objects $\mathcal{O}$, set of probe functions $\mathcal{F}$ representing object features, $\sim_{B_r}$indiscernibility relation B_r defined relative to $B_r \subseteq B \subseteq \mathcal{F}$, and collection of partitions (families of neighbourhoods) $N_r(B)$. This relation $\sim_{B_r}$defines a partition of $\mathcal{O}$ into non-empty, pairwise disjoint subsets that are equivalence classes denoted by $[x]_{B_r}$, where $[x]_{B_r} = \{x' \in \mathcal{O} \mid x \sim_{B_r} x'\}$. These classes form a new set called the quotient set $\mathcal{O}/\sim_{B_r}$, where $\mathcal{O}/\sim_{B_r} = \left\{[x]_{B_r} \mid x \in \mathcal{O}\right\}$. In effect, each choice of probe functions B_r defines a partition $\xi_{\mathcal{O},B_r}$ on a set of objects $\mathcal{O}$, namely, $\xi_{\mathcal{O},B_r} = \mathcal{O}/\sim_{B_r}$. Let we consider $X \subseteq \mathcal{O}$, then upper approximation of X defined by

$$N_r(B)^* X = \bigcup_{[x]_{B_r} \cap X \neq \varnothing} [x]_{B_r}$$

and lower approximation of X defined by

$$N_r(B)_* X = \bigcup_{[x]_{B_r} \subseteq X} [x]_{B_r}$$

(See [14, 19]).

Theorem 2.3 ([14])**.** *Let $(\mathcal{O}, \mathcal{F}, \sim_{B_r}, N_r(B))$ be a weak nearness approximation space and $X, Y \subset \mathcal{O}$, then the following statements hold:*

(1) $N_r(B)_* X \subseteq X \subseteq N_r(B)^* X$,
(2) $N_r(B)^* (X \cup Y) = (N_r(B)^* X) \cup (N_r(B)^* Y)$,
(3) $X \subseteq Y$ *implies* $N_r(B)^* X \subseteq N_r(B)^* Y$,
(4) $N_r(B)^* (X \cap Y) \subseteq (N_r(B)^* X) \cap (N_r(B)^* Y)$,

Definition 2.4 ([5]). Let $(\mathcal{O}, \mathcal{F}, \sim_{B_r}, N_r(B))$ be a weak nearness approximation space and $S \subset \mathcal{O}$. If the following properties are satisfied, then S is called a semigroup on weak approximate approximation space $\mathcal{O}$, or in short, a nearness semigroup.
(i) $x \cdot y \in N_r(B)^* S$ for all $x, y \in S$,
(ii) $(x \cdot y) \cdot z = x \cdot (y \cdot z)$ property holds in $N_r(B)^* S$ for all $x, y, z \in S$.

Definition 2.5 ([7]). $(S, \cdot)$ is called a nearness monoid, if S is a nearness semigroup in which there exists an element $e \in N_r(B)^* S$ satisfying $x \cdot e = e \cdot x = x$, for all $x \in S$.

Definition 2.6 ([7]). A nearness monoid $(S, \cdot)$ ($(S, +)$) is called a commutative (abelian), if $x \cdot y = y \cdot x$ ($x + y = y + x$), for all $x, y \in S$.

For other notions and definitions not mentioned in this paper, the readers are referred to [1, 2, 3, 4, 5, 6, 7, 8, 13, 19, 20, 22].

3. Nearness Hemirings

Definition 3.1 ([7]). A subset H of the weak near approximation space $\mathcal{O}$ is called a hemiring on $\mathcal{O}$, if the following properties are satisfied:
(NHR_1) $(H, +)$ is an abelian monoid on $\mathcal{O}$ with identity element 0_H,
(NHR_2) $(H, \cdot)$ is a semigroup on $\mathcal{O}$,
(NHR_3) for all $x, y, z \in H$,

$$x \cdot (y + z) = x \cdot y + x \cdot z \ \text{ and } (x + y) \cdot z = x \cdot z + y \cdot z$$

properties hold in $N_r(B)^* H$,
(NHR_4) for all $x \in H$,

$$0_H \cdot x = 0_H = x \cdot 0_H$$

hold in $N_r(B)^* S$.

Definition 3.2. Let H be a hemiring on $\mathcal{O}$, $B_r \subseteq \mathcal{F}$ where $r \leq | B |$and $B \subseteq \mathcal{F}$, $\sim_{B_r}$be an indiscernibility relation on $\mathcal{O}$. Then $\sim_{B_r}$ is called a congruence indiscernibility relation on nearness hemiring H, if $x \sim_{B_r} y$ and $\gamma \sim_{B_r} \beta$, where $x, y \in H$ implies $(x+b) \sim_{B_r} (y+b)$, $(b+x) \sim_{B_r} (b+y)$ and $(x \cdot b) \sim_{B_r} (y \cdot b)$, $(b \cdot x) \sim_{B_r} (b \cdot y)$ for all $a, b \in H$.

We will give the following Lemmas, with proofs that are symmetric to the proofs of [7, Lemma 3.8] and [7, Lemma 3.10], respectively.

Lemma 3.3. *Let H be a nearness hemiring. If $\sim_{B_r}$is a congruence indiscernibility relation on H, then $[x]_{B_r} + [y]_{B_r} \subseteq [x + y]_{B_r}$, and $[x]_{B_r} \cdot [y]_{B_r} \subseteq [x \cdot y]_{B_r}$ for all $x, y \in H$.*

Definition 3.4. Let H be a hemiring on $\mathcal{O}$, $B_r \subseteq \mathcal{F}$ where $r \leq \mid B \mid$and $B \subseteq \mathcal{F}$, $\sim_{B_r}$ be an indiscernibility relation on $\mathcal{O}$. Then, $\sim_{B_r}$ is called a complete congruence indiscernibility relation on nearness hemiring H, if $[x]_{B_r} + [y]_{B_r} = [x+y]_{B_r}$ and $[x]_{B_r} \cdot [y]_{B_r} = [x \cdot y]_{B_r}$ for all $x, y \in H$.

Let H be a nearness hemiring. Let $X + Y = \{x + y \mid x \in X, \text{ and } y \in Y\}$ and $X \cdot Y = \{\sum_{finite} x_i y_i \mid x_i \in X \text{ , and } y_i \in Y\}$, where X and Y are subsets of H.

Lemma 3.5. *Let H be a nearness hemiring, and $\sim_{B_r}$be a congruence indiscernibility relation on H. The following properties hold*:
(1) *if $X, Y \subseteq H$, then $(N_r(B)^* X) + (N_r(B)^* Y) \subseteq N_r(B)^* (X+Y)$,*
(2) *if $X, Y \subseteq H$, then $(N_r(B)^* X) \cdot (N_r(B)^* Y) \subseteq N_r(B)^* (X \cdot Y)$.*

Definition 3.6. Let H be a nearness hemiring and let A be a non-empty subset of H.
(i) A is called a sub-hemiring of H, if $A + A \subseteq N_r(B)^* A$ and $A \cdot A \subseteq N_r(B)^* A$.
(ii) A is called an upper-near sub-hemiring of H, if $(N_r(B)^* A) + (N_r(B)^* A) \subseteq N_r(B)^* A$ and $(N_r(B)^* A) \cdot (N_r(B)^* A) \subseteq N_r(B)^* A$.

Theorem 3.7. *Let H be a nearness hemiring. The following properties hold*:
(1) *if $\varnothing \neq A \subseteq H$, $A+A \subseteq A$ and $A \cdot A \subseteq A$, then A is an upper-near sub-hemiring of H,*
(2) *if A is a sub-hemiring of H, $N_r(B)^* (N_r(B)^* A) = N_r(B)^* A$, then A is an upper-near sub-hemiring of H.*

Proof. The proof is straightforward and similar to the proof of [7, Theorem 3.14]. □

Definition 3.8. Let H be a nearness hemiring, and A a sub-hemiring of H.
(i) A is called a right (left) ideal of H if $A \cdot S \subseteq N_r(B)^* A$ $(S \cdot A \subseteq N_r(B)^* A)$.
(ii) A is called an upper-near right (left) ideal of H if $(N_r(B)^* A) \cdot H \subseteq N_r(B)^* A$ $(H \cdot (N_r(B)^* A) \subseteq N_r(B)^* A)$.

We will give the following Theorems, which are symmetric to the proofs of [7, Theorem 3.16] and [7, Theorem 3.17], respectively.

Theorem 3.9. *Let H be a nearness hemiring. The following properties hold*:
(1) *if $\varnothing \neq A \subseteq H$, $A + A \subseteq A$ and $A \cdot A \subseteq A$, then A is an upper-near right (left) ideal of H,*
(2) *if A is a right (left) ideal of H, and $N_r(B)^* (N_r(B)^* A) = N_r(B)^* A$, then A is an upper-near right (left) ideal of H.*

Theorem 3.10. *Let $\{A_i \mid i \in I\}$ be a set of ideals of nearness hemiring H, where I is an arbitrary index set.*
(1) *If $N_r(B)^*(\bigcap_{i\in I} A_i) = \bigcap_{i\in I} N_r(B)^* A_i$, then $\bigcap_{i\in I} A_i$ is an ideal of H.*
(2) *$\bigcup_{i\in I} A_i$ is an ideal of H.*

4. Prime ideals of the nearness Hemirings

Definition 4.1. Let H be a nearness hemiring and let A, A_1, A_2 and P be ideals of H. P is called a prime (resp. semiprime) ideal of H, if $A_1A_2 \subseteq N_r(B)^*P$ ($A^2 = AA \subseteq N_r(B)^*P$) implies $A_1 \subseteq P$ or $A_2 \subseteq P$ (resp. $A \subseteq P$).

Definition 4.2. Let H be a nearness hemiring and let A, A_1, A_2 and P be ideals of H. P is called an upper-near prime (resp. an upper-near semiprime) ideal of H, if $(N_r(B)^*A_1)(N_r(B)^*A_2) \subseteq N_r(B)^*P$ (resp. $(N_r(B)^*A)(N_r(B)^*A) \subseteq N_r(B)^*P$) implies $N_r(B)^*A_1 \subseteq P$ or $N_r(B)^*A_2 \subseteq P$ (resp. $N_r(B)^*A \subseteq P$).

Theorem 4.3. *Let H be a nearness hemiring, $\sim_{B_r}$ be a congruence indiscernibility relation on H, and let A_1, A_2 and P be prime ideals of S such that $N_r(B)^*(N_r(B)^*A_1) = N_r(B)^*A_1$, $N_r(B)^*(N_r(B)^*A_2) = N_r(B)^*A_2$ and $N_r(B)^*(N_r(B)^*P) = N_r(B)^*P$, respectively. If $(N_r(B)^*A_1)(N_r(B)^*A_2) \subseteq N_r(B)^*P$, then P is an upper-near prime ideal of H.*

Proof. Since P is a prime ideal of H such that $N_r(B)^*(N_r(B)^*P) = N_r(B)^*P$, P is an upper-near-ideal of H by Theorem 3.9 (2). Suppose that $(N_r(B)^*A_1)(N_r(B)^*A_2) \subseteq N_r(B)^*P$ such that $N_r(B)^*A_1 \subsetneqq P$ or $N_r(B)^*A_2 \subsetneqq P$. Then there exists an element $x \in N_r(B)^*A_1$ such that $x \notin P$ and $y \in N_r(B)^*A_2$ such that $y \notin P$. From here, $[x]_{B_r} \cap A_1 \neq \varnothing$ and $[y]_{B_r} \cap A_2 \neq \varnothing \Rightarrow a_1 \in [x]_{B_r}, a_1 \in A_1$ and $a_2 \in [x]_{B_r}, a_2 \in A_2 \Rightarrow x \sim_{B_r} a_1, a_2 \in A_1$ and $y \sim_{B_r} a_2, a_2 \in A_2$. Since $\sim_{B_r}$ is a congruence indiscernibility relation on H and A_1, and A_2 are ideals of H insert space to obtain, we have $(xy) \sim_{B_r} (a_1a_2), a_1a_2 \in (N_r(B)^*A_1) \cap (N_r(B)^*A_2)$. Thus $[xy]_{B_r} \cap ((N_r(B)^*A_1) \cap (N_r(B)^*A_2)) \neq \varnothing \Rightarrow xy \in N_r(B)^*((N_r(B)^*A_1) \cap (N_r(B)^*A_2)) \Rightarrow xy \in N_r(B)^*((N_r(B)^*A_1)$ and $xy \in N_r(B)^*(N_r(B)^*A_2))$ by Theorem 2.3 (4). From the hypothesis, we get $xy \in N_r(B)^*A_1$ and $xy \in N_r(B)^*A_2 \Rightarrow (xy)^2 = (xy)(xy) \in (N_r(B)^*A_1)(N_r(B)^*A_2)$. So $(xy)^2 \in N_r(B)^*P$. Since P is a prime ideal, $xy \in P$, which is a contradiction. Hence we have $N_r(B)^*A_1 \subseteq P$ or $N_r(B)^*A_2 \subseteq P$. □

Theorem 4.4. *Let H be a nearness hemiring, $\sim_{B_r}$ be a congruence indiscernibility relation on H, A and P be semiprime ideals of H such that $N_r(B)^*(N_r(B)^*A) = N_r(B)^*A$ and $N_r(B)^*(N_r(B)^*P) = N_r(B)^*P$, respectively. If $(N_r(B)^*A)(N_r(B)^*A) \subseteq N_r(B)^*P$, then P is an upper-near semiprime ideal of H.*

Proof. The proof is straightforward and similar to the proof of Theorem 4.3. □

Let A be a non-empty subset of nearness hemiring H and $h \in H$. Let $hA = \{\sum\limits_{finite} ha_i \mid a_i \in A\}$.

Theorem 4.5. *Let H be a nearness hemiring, $\sim_{B_r}$ be a congruence indiscernibility relation on H, and let $a \in H$. Then aH is a right ideal of H.*

Proof. Let $x, y \in aH$. In this case, $x = \sum\limits_{finite} ah_i$; $h_i \in H$ and $y = \sum\limits_{finite} ah_i^{'}$; $h_i^{'} \in H$. Then $x + y = (\sum\limits_{finite} ah_i) + (\sum\limits_{finite} ah_i^{'}) = \sum\limits_{finite} as_i \in a(N_r(B)^*H)$ for all $s_i \in H$. Thus there exist $z \in N_r(B)^*H$ such that $x + y = az$ for any $a \in H$ and $z \in N_r(B)^*H \Rightarrow [z]_{B_r} \cap H \neq \varnothing \Rightarrow c \in [z]_{B_r}, c \in H \Rightarrow z \sim_{B_r} c, c \in H$. Since $\sim_{B_r}$ is a congruence indiscernibility relation on H, we get $a\gamma z \sim_{B_r} a\gamma c, c \in H, \gamma \in \Gamma \Rightarrow a\gamma c \in [a\gamma z]_{B_r}$

and $ac \in aH \Rightarrow [az]_{B_r} \cap (aH) \neq \varnothing$. So we obtain $x+y = az \in N_r(B)^*(aH)$, namely, $(aH) + (aH) \subseteq N_r(B)^*(aH)$.

Now, let $x \in aH$ and $h \in H$. Then $x = \sum\limits_{finite} ah'_i$; $h'_i \in H$. Thus we get $xh = (\sum\limits_{finite} ah'_i)h = \sum\limits_{finite} a(h'_i h) \in a(N_r(B)^*H)$. So there exists $b \in N_r(B)^*H$ such that $xh = ab$ for all $h \in H$ and $b \in N_r(B)^*H \Rightarrow [b]_{B_r} \cap H \neq \varnothing \Rightarrow z \in [b]_{B_r}, z \in H \Rightarrow b \sim_{B_r} z, z \in H$. Since $\sim_{B_r}$ is a congruence indiscernibility relation on H, we obtain $ab \sim_{B_r} az, z \in H \Rightarrow az \in [ab]_{B_r}$ and $az \in aH \Rightarrow [ab]_{B_r} \cap (aH) \neq \varnothing$. Hence we have $xh = ab \in N_r(B)^*(aH)$. Therefore we obtain $(aH)H \subseteq N_r(B)^*(aH)$. □

Theorem 4.6. *Let H be a nearness hemiring and let $\{P_i \mid i \in I\}$ be a set of prime (resp. semiprime) ideals of H, where I is an arbitrary index set.*

(1) If $N_r(B)^*(\bigcap\limits_{i\in I} P_i) = \bigcap\limits_{i\in I} N_r(B)^*P_i$, then $\bigcap\limits_{i\in I} P_i$ is a prime (resp. semiprime) ideal of H.

(2) If $P_1 \subseteq P_2 \subseteq P_3 \subseteq ...$, then $\bigcup\limits_{i\in I} P_i$ is a prime (resp. semiprime) ideal of H.

Proof. (1) From Theorem 3.10 (1), we get that $\bigcap\limits_{i\in I} P_i$ is an ideal of H. Now, let $A_1A_2 \subseteq N_r(B)^*(\bigcap\limits_{i\in I} P_i)$ for any ideals A_1 and A_2 of H. Then $A_1A_2 \subseteq \bigcap\limits_{i\in I} N_r(B)^*P_i$ by hypothesis, and we have $A_1A_2 \subseteq N_r(B)^*P_i$ for all $i \in I$. Since P_i are prime ideals of H for all $i \in I$, we get $A_1 \subseteq P_i$ or $A_2 \subseteq P_i$ for all $i \in I$. In this case, $A_1 \subseteq \bigcap\limits_{i\in I} P_i$ or $A_2 \subseteq \bigcap\limits_{i\in I} P_i$.

(2) $\bigcup\limits_{i\in I} P_i$ is an ideal of H, by Theorem 3.10 (2). $A_1A_2 \subseteq N_r(B)^*(\bigcup\limits_{i\in I} P_i)$ for any ideals A_1 and A_2 of H. Then $A_1A_2 \subseteq \bigcup\limits_{i\in I} N_r(B)^*P_i$, by Theorem 2.3 (2). Thus there is at least one $i_0 \in I$ such that $A_1A_2 \subseteq N_r(B)^*P_{i_0}$, by hypothesis. Since P_{i_0} is prime ideal of H for $i_0 \in I$, we have $A_1 \subseteq P_{i_0}$ or $A_2 \subseteq P_{i_0}$ for $i_0 \in I$. So $A_1 \subseteq \bigcup\limits_{i\in I} P_i$ or $A_2 \subseteq \bigcup\limits_{i\in I} P_i$. □

Theorem 4.7. *Let H be a nearness hemiring, P be a right ideal of H such that $N_r(B)^*(N_r(B)^*P) = N_r(B)^*P$ and $a, b \in H$. If $aHb \subseteq N_r(B)^*P$ implies $a \in P$ or $b \in P$, then P is a prime right ideal of H.*

Proof. Let A_1 and A_2 be any two right ideals of H such that $A_1A_2 \subseteq N_r(B)^*P$ and $A_1 \nsubseteq P$. Then there exist an element $a_1 \in A_1$ such that $a_1 \notin P$. For any $a_2 \in A_2$, we have $a_1Ha_2 = (a_1H)a_2 \subseteq (N_r(B)^*A_1)a_2$. On the other hand, let $x \in (N_r(B)^*A_1)a_2$ such that $x = \sum\limits_{finite} y_ia_2$; $y_i \in N_r(B)^*A_1$, $a_2 \in A_2$, $1 \leq i \leq n$. Then $y_i \in N_r(B)^*A_1 \Rightarrow [y_i]_{B_r} \cap A_1 \neq \varnothing \Rightarrow c \in [y_i]_{B_r}$, $c \in A_1 \Rightarrow y_i \sim_{B_r} c$, $c \in A_1$, $1 \leq i \leq n$. Since $\sim_{B_r}$ is a congruence indiscernibility relation on H, we get $(y_ia_2) \sim_{B_r} (ca_2)$, $ca_2 \in A_1A_2 \subseteq N_r(B)^*P$, $1 \leq i \leq n$. Because of that P is a right ideal of H such that $N_r(B)^*(N_r(B)^*P) = N_r(B)^*P$, we get $(\sum\limits_{finite} y_ia_2) \sim_{B_r} (\sum\limits_{finite} ca_2)$, $\sum\limits_{finite} ca_2 \in N_r(B)^*P$, $1 \leq i \leq n$. Thus $\sum\limits_{finite} ca_2 \in [\sum\limits_{finite} y_ia_2]_{B_r}$ and $\sum\limits_{finite} ca_2 \in N_r(B)^*P \Rightarrow [\sum\limits_{finite} y_ia_2]_{B_r} \cap (N_r(B)^*P) \neq \varnothing \Rightarrow [x]_{B_r} \cap (N_r(B)^*P) \neq \varnothing$. So we obtain $x \in N_r(B)^*(N_r(B)^*P) = N_r(B)^*P$, namely, $a_1Ha_2 \subseteq N_r(B)^*P$. Hence we have $a_2 \in P$, by the hypothesis, i.e. , $A_2 \subseteq P$. □

Theorem 4.8. *Let H be a nearness hemiring, $\sim_{B_r}$ be a congruence indiscernibility relation on H, and let P be a right ideal of H such that $N_r(B)^*(N_r(B)^*P) = N_r(B)^*P$ and $a \in H$. If $aHa \subseteq N_r(B)^*P$ implies $a \in P$, then P is a semiprime right ideal of H.*

Definition 4.9. Let H be a nearness hemiring and let A_1, A_2 and P be ideals of H. P is called an irreducible (resp. a strongly irreducible) ideal of H, if $A_1 \cap A_2 = N_r(B)^*P$ (resp. $A_1 \cap A_2 \subseteq N_r(B)^*P$) implies $A_1 = P$ or $A_2 = P$ (resp. $A_1 \subseteq P$ or $A_2 \subseteq P$).

Theorem 4.10. *Let H be a nearness hemiring and let $\{A_i \mid i \in I\}$ be a set of ideals of H, where I is an arbitrary index set. If $N_r(B)^*(\bigcap\limits_{i\in I} A_i) = \bigcap\limits_{i\in I} N_r(B)^*A_i$, then every strongly irreducible and semiprime ideal of H is a prime ideal of H.*

Proof. Let $N_r(B)^*(\bigcap\limits_{i\in I} A_i) = \bigcap\limits_{i\in I} N_r(B)^*A_i$ and P be a strongly irreducible and semiprime ideal of H. Let $A_1A_2 \subseteq N_r(B)^*P$ for any ideals A_1 and A_2 of H. Then $A_1 \cap A_2$ is an ideal of H, by Theorem 3.10 (1). Thus $(A_1 \cap A_2)^2 = (A_1 \cap A_2)(A_1 \cap A_2) \subseteq A_1A_2 \subseteq N_r(B)^*P \Rightarrow (A_1 \cap A_2)^2 \subseteq N_r(B)^*P$. Since P is a semiprime ideal of H, we get $A_1 \cap A_2 \subseteq P$. So $A_1 \cap A_2 \subseteq N_r(B)^*P$, by Theorem 2.3 (1). Hence we obtain $A_1 \subseteq P$ or $A_2 \subseteq P$, for P is a strongly irreducible ideal of H. □

Acknowledgements. The author would like to thank the anonymous reviewers for their careful reading of this article and for their helpful comments.

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