

Intuitionistic fuzzy ideal topological groups

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ABSTRACT. In this paper, we will introduce a new class of topological groups called intuitionistic fuzzy ideal topological groups by depended on an intuitionistic fuzzy topological groups (X, τ) .

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1. INTRODUCTION

The definition of fuzzy sets introduced by Zadeh [1]. Foster [2] introduced the concept of a fuzzy topological group using the Lowen's definition of a fuzzy topological space (See [3]). Ma and Yu [4, 5] changed the definition of a fuzzy topological group in order to make sure that an ordinary topological group is a special case of a fuzzy topological group. Soft sets is a more detailed study of uncertainty on fuzzy sets. Nazmul and Samanta [6] introduced fuzzy soft topological groups.

Atanassov [7] intraduced the notion of intuitionistic fuzzy sets. In many applications, the intuitionist fuzzy sets are important and useful fuzzy sets. Atanassov [8, 9] in 1994 and 1999 proved that the intuitionistic fuzzy sets contain the degree of affiliation and the degree of non-affiliation, and therefore, the intuitionistic fuzzy sets have become more relevant and applicable. In 2001 and 2004, Szmidi and Kacprzyk [10, 11] showed that intuitionist fuzzy sets are so useful in situations where it seems extremely difficult to define a problem through a membership function. Çoker and Demirci [12, 13] introduced the notion of intuitionistic fuzzy point and Çoker [14] defined an intuitionistic fuzzy topology and some of its properties. Kuratowski first proposed the concept of an ideal topological space [15]. Intuitionistic fuzzy ideal topological space was introduced by Salama and Alblowi [16]. Hur et al. [17, 18, 19, 20] studied various properties of intuitionistic fuzzy subgroupoids,

intuitionistic fuzzy subgroups, intuitionistic fuzzy sub-rings and intuitionistic fuzzy topological groups.

In this paper, we will introduce a new class of topological groups called intuitionistic fuzzy ideal topological groups by depended on an intuitionistic fuzzy topological groups (X, τ) .

2. PRELIMINARIES

Definition 2.1 ([7]). Let $X \neq \emptyset$. Then a function $E = (\mu_E, \nu_E) : X \rightarrow [0, 1] \times [0, 1]$ is said to be *intuitionistic fuzzy set* in X (ifs, for short). If $0 \leq \mu_E(x) + \nu_E(x) \leq 1$ for every $x \in X$, where the function $\mu_E : X \rightarrow [0, 1]$ is the degree of membership ($\mu_E(x)$) and $\nu_E : X \rightarrow [0, 1]$ is the degree of nonmembership ($\nu_E(x)$) for every $x \in X$. We will denoted the set of all ifs in X by $IF(X)$.

Definition 2.2 ([7]). Let $B, E \in IFX$ be an ifs. Then

- (i) $B \subseteq E$ iff $\mu_B \leq \mu_E$ and $\nu_B \geq \nu_E$,
- (ii) $B = E$ iff $B \subseteq E$ and $E \subseteq B$,
- (iii) $B^c = (\nu_B, \mu_B)$,
- (iv) $B \cap E = (\mu_B \wedge \mu_E, \nu_B \vee \nu_E)$,
- (v) $B \cup E = (\mu_B \vee \mu_E, \nu_B \wedge \nu_E)$,
- (vi) $0_{\sim} = (0, 1)$ and $1_{\sim} = (1, 0)$.

Definition 2.3 ([12, 13]). Let $X \neq \emptyset$ and let $x \in X$. If $t \in (0, 1]$ and $r \in [0, 1)$ are two fixed real numbers such that $t + r \leq 1$, then

$$x_{(t,r)} = \{ \langle x, x_{(t)}, 1 - x_{(r)} \rangle : x \in X \}$$

is said to be an *intuitionistic fuzzy point* (ifp, for short) in X .

Definition 2.4 ([14]). A subclass τ is said to be an *intuitionistic fuzzy topology* on X , if

- (i) $0_{\sim}, 1_{\sim} \in \tau$,
- (ii) $B_1 \cap B_2 \in \tau$ for every $B_1, B_2 \in \tau$,
- (iii) $\bigcup_{i \in \Gamma} B_i \in \tau$ for every $B_i \in \tau$.

The pair (X, τ) is said to be an *intuitionistic fuzzy topological space* (ifts, for short). Any ifs B in τ is said to be an *intuitionistic fuzzy open set*, and the complement B^c of an intuitionistic fuzzy open set B is said to be an *intuitionistic fuzzy closed set*.

Definition 2.5 ([14]). Let (X, τ) be an ifts and let $B \in IF(X)$. Then the *intuitionistic fuzzy interior* and the *intuitionistic fuzzy closure* of B in (X, τ) defined as

$$int(B) = \bigcup \{U : U \subseteq B, U \in \tau\}$$

and

$$cl(B) = \bigcap \{F : B \subseteq F, F^c \in \tau\},$$

respectively.

Definition 2.6 ([14]). Let $B, E \in IF(X)$. Then B is said to be *quasi-coincident with E* (written BqE), if there is $x \in X$ such that $\mu_B(x) > \nu_E(x)$ or $\nu_B(x) < \mu_E(x)$.

Definition 2.7 ([14]). Let (X, τ) be an ifts and let $B \in IF(X)$. Then B is called an *intuitionistic fuzzy neighbourhood* of an intuitionistic fuzzy point $x_{(t,r)}$, if there is $V \in \tau$ with $x_{(t,r)} \in V \subseteq B$. The collection $N(x_{(t,r)})$ of all neighbourhood of $x_{(t,r)}$ is said to be the *intuitionistic fuzzy neighbourhood system* of $x_{(t,r)}$.

Definition 2.8 ([21]). Let X be a group and let $G \in IF(X)$. Then G is said to be an *intuitionistic fuzzy subgroup* (ifsg, for short) in X , if it satisfies the following conditions: for every $x, y \in X$,

- (i) $\mu_G(xy) \geq \min\{\mu_G(x), \mu_G(y)\}$,
- (ii) $\mu_G(x^{-1}) \geq \mu_G(x)$,
- (iii) $\nu_G(xy) \leq \max\{\nu_G(x), \nu_G(y)\}$,
- (iv) $\nu_G(x^{-1}) \leq \nu_G(x)$.

Proposition 2.9. G is an ifsg in X iff for all $x, y \in X$,

$$\mu_G(xy^{-1}) \geq \min\{\mu_G(x), \mu_G(y)\} \text{ and } \nu_G(xy^{-1}) \leq \max\{\nu_G(x), \nu_G(y)\}.$$

Proof. See [21]. □

Definition 2.10 ([22]). Let X be a group and let $G \in IF(X)$ an ifs in X . Then G is said to be an *intuitionistic fuzzy normal subgroup* (ifnsg, for short) in X , if

- (i) $\mu_G(xy) = \mu_G(yx)$,
- (ii) $\nu_G(xy) = \nu_G(yx)$ for every $x, y \in X$.

Proposition 2.11. G is an ifnsg in X iff for every $x \in G$ and $g \in X$,

- (1) $\mu_G(g^{-1}xg) = \mu_G(x)$,
- (2) $\nu_G(g^{-1}xg) = \nu_G(x)$.

Proof. See [22]. □

Definition 2.12 ([23]). Let (X, τ) an ifts, let G be an ifsg in X and let G be given with due ift τ . Then G is an *intuitionistic fuzzy topological group* (iftg, for short) in X , if it satisfies the following conditions:

- (i) the operation $\gamma : (x, y) \rightarrow xy$ of $(G, \tau) \times (G, \tau) \rightarrow (G, \tau)$ is intuitionistic fuzzy continuous.
- (ii) the operation $\xi : x \rightarrow x^{-1}$ of $(G, \tau) \rightarrow (G, \tau)$ is intuitionistic fuzzy continuous.

Proposition 2.13. Let (X, τ) an ifts. An ifsg G in X is an iftg iff the operation $\delta : (x, y) \rightarrow xy^{-1}$ of $(G, \tau) \times (G, \tau) \rightarrow (G, \tau)$ is intuitionistic fuzzy continuous.

Proof. See [23]. □

Definition 2.14 ([16]). Let $I \neq \emptyset$ be a family of intuitionistic fuzzy sets of X . Then I is said to be an *intuitionistic fuzzy ideal* on X , if it satisfies the following conditions:

- (i) if $B \in I$ and $E \in IF(X)$ such that $E \subseteq B$, then $E \in I$,
- (ii) if $B, E \in I$, then $B \cup E \in I$.

Let (X, τ) be an intuitionistic fuzzy topological space. Then an intuitionistic fuzzy ideal I on X is called an *intuitionistic fuzzy ideal topological space* (shortly, ifts) is defined by (X, τ, I) . The elements of (X, τ, I) are said to be *intuitionistic fuzzy-I-open sets* and the *complement* of intuitionistic fuzzy open sets are said to be *fuzzy-I-closed sets*.

Definition 2.15 ([16]). Let (X, τ, I) be an ifts and let $B \in IF(X)$. Then the intuitionistic fuzzy local function $B^*(I, \tau)$ of B is the union of all intuitionistic fuzzy points $x_{(t,r)}$, i.e.,

$$B^*(I, \tau) = \bigcup \{x_{(t,r)} \in X : B \cap V \notin I, \text{ for every } V \in N(x_{(t,r)})\}.$$

Definition 2.16 ([16]). Let B be an intuitionistic fuzzy set in an ifts (X, τ, I) . Then the closure operator for B define as $Cl^*(B) = B \cup B^*$. Also the operator Cl^* for a topology $\tau^*(I)$ finer than τ and a basis $\beta(I, \tau)$ for $\tau^*(I)$ describe as $\beta(I, \tau) = \{V - E : V \in \tau, E \in I\}$.

Definition 2.17 ([16]). Let (X, τ, I) be an ifts. Then I is called compatible with τ , and denoted by $I \sim \tau$, if for each $B \in IF(X)$ and each $x_{(t,r)} \in B$, there is $V \in N(x_{(t,r)})$ such that $V \cap B \in I$, then $B \in I$. Also I is said to be τ -boundary, if $I \cap \tau = 0_{\sim}$.

Definition 2.18 ([16]). Let (X, τ, I) be an ifts. An operator $\Psi : IF(X) \rightarrow \tau$ is defined as follows: for each $B \in I^X$,

$$\Psi(B) = \{x_{(t,r)} : \text{there is } V \in N(x_{(t,r)}) \text{ such that } V - B \in I\}.$$

Observe that $\Psi(B) = X - (X - B)^*$.

3. INTUITIONISTIC FUZZY IDEAL TOPOLOGICAL GROUPS

Definition 3.1. An ifs B in an ifts (X, τ, I) is called an I -neighbourhood of x_t , if there is an intuitionistic fuzzy- I -open set V such that $x_t \in V \subseteq B$.

Definition 3.2. Let (X, τ, \cdot) be an iftg and let I be an intuitionistic fuzzy ideal on X . Then (X, τ, I, \cdot) is said to be an intuitionistic fuzzy ideal topological group (ifitg, for short), if for every ifp $x_{(t,r)}, y_{(t,r)}$ in X and every $M \in N(x_{(t,r)}y_{(t,r)}^{-1})$ in X , there is intuitionistic fuzzy- I -open sets U and V , $U \in N(x_{(t,r)}), V \in N(y_{(t,r)})$ such that $UV^{-1} \subseteq M$.

Definition 3.3. Let (X, τ, I, \cdot) be an ifitg. Then I is said to be

- (i) *left translation*, if for every ifp $x_{(t,r)}$ in X , $x_{(t,r)}I \subseteq I$,
- (ii) *right translation*, if for every ifp $x_{(t,r)}$ in X , $Ix_{(t,r)} \subseteq I$,

where $x_{(t,r)}I = \{x_{(t,r)}E : E \in I\}$ and $Ix_{(t,r)} = \{Ex_{(t,r)} : E \in I\}$.

It is obvious that if I is left [resp. right] translation, then $x_{(t,r)}I = I$ [resp. $Ix_{(t,r)} = I$] for every ifp $x_{(t,r)}$ in X .

Corollary 3.4. Let (X, τ, I, \cdot) be an ifitg. If I is left or right translation, $I \neq IF(X)$ and $I \sim \tau$, then I is τ -boundary.

Proposition 3.5. Let (X, τ, I, \cdot) be an ifitgp, let A, B be an ifs and let $x_{(t,r)}$ be an ifp in X .

- (1) If A is intuitionistic fuzzy- I -open, then $Ax_{(t,r)}$ and $x_{(t,r)}A$ are intuitionistic fuzzy- I -open.
- (2) If A is intuitionistic fuzzy- I -open, then AB and BA are intuitionistic fuzzy- I -open.
- (3) If A is intuitionistic fuzzy- I -closed, then $Ax_{(t,r)}$ and $x_{(t,r)}A$ are intuitionistic fuzzy- I -closed.

(4) If A is intuitionistic fuzzy- I -closed, then AB and BA are intuitionistic fuzzy- I -closed.

(5) A is intuitionistic fuzzy- I -open iff A^{-1} is intuitionistic fuzzy- I -open.

(6) A is intuitionistic fuzzy- I -closed iff A^{-1} is intuitionistic fuzzy- I -closed.

Proof. It is clear from Definition 2.2. □

Theorem 3.6. Let (X, τ, I, \cdot) be an ifitg with $I \sim \tau$. Let $Q \in \mathcal{U}(X) = \{A \in I^X : \text{there is a } B \in \beta(I, \tau) \text{ such that } B \subseteq A\}$ and let $S \in IF(X) - I$. Let U and V are fuzzy- I -open sets such that $U \cap S^* \neq 0_\sim \neq V \cap \text{int}(Q^*) \cap \Psi(Q)$. Let $A = U \cap S \cap S^*$ and $B = V \cap Q \cap \text{int}(Q^*) \cap \Psi(Q)$.

(1) If I is right translation, then $A^{-1}B$ is a nonempty intuitionistic fuzzy- I -open subset of $S^{-1}Q$.

(2) If I is left translation, then BA^{-1} is a nonempty intuitionistic fuzzy- I -open subset of QS^{-1} .

Proof. (1) Let $f_{x_{(a_1, a_2)}} : IF(X) \rightarrow IF(X)$ define as $f_{x_{(a_1, a_2)}}(x_{(t, r)}) = x_{(a_1, a_2)}^{-1}x_{(t, r)}$ for any $x_{(a_1, a_2)} \in A$ and let $\mathcal{F} = \{f_{x_{(a_1, a_2)}} : x_{(a_1, a_2)} \in A\}$. From [?], we have

$$U \cap S^* \subseteq (U \cap S)^* = (U \cap S \cap S^*)^*.$$

Since $I \sim \tau$ and $U \cap S^* = 0_\sim$, $A \neq 0_\sim$ and $\mathcal{F} \neq 0_\sim$. Then $A \subseteq A^*$ and each $f_{x_{(a_1, a_2)}}$ is an intuitionistic fuzzy homomorphism. Let $D = V \cap \text{int}(Q^*) \cap \Psi(Q)$. If $D \cap \mathcal{F}^{-1}(y_{(t, r)}) \notin I$ for each $y_{(t, r)} \in \mathcal{F}(D)$, then $\mathcal{F}(D \cap Q) = \mathcal{F}(B) = A^{-1}B$ is nonempty intuitionistic fuzzy- I -open subset of $S^{-1}Q$.

Now let $y_{(t, r)} \in \mathcal{F}(D)$. Then $y_{(t, r)} = x_{(a_1, a_2)}^{-1}x_{(t, r)}$ for some $x_{(a_1, a_2)} \in A$ and $x_{(t, r)} \in D \rightarrow \mathcal{F}^{-1}(y_{(t, r)}) = Ax_{(a_1, a_2)}^{-1}x_{(t, r)}$. Thus we have

$$\begin{aligned} x_{(t, r)} \in Ax_{(a_1, a_2)}^{-1}x_{(t, r)} &\subseteq A^*x_{(a_1, a_2)}^{-1}x_{(t, r)} \subseteq (Ax_{(a_1, a_2)}^{-1}x_{(t, r)})^* \\ &= (\mathcal{F}^{-1}(y_{(t, r)}))^* \rightarrow G \cap \mathcal{F}^{-1}(y_{(t, r)}) \notin I \text{ for each } G \in N(x_{(t, r)}). \end{aligned}$$

Especially, $D \in N(x_{(t, r)}) \rightarrow D \cap \mathcal{F}^{-1}(y_{(t, r)}) \notin I$. So $A^{-1}B$ is nonempty intuitionistic fuzzy- I -open subset of $S^{-1}Q$.

(2) The proof is similar to (1). □

Theorem 3.7. Let (X, τ, I, \cdot) be an ifitg with $I \sim \tau$. Let $Q \in \mathcal{U}(X)$ and let $S \in IF(X) - I$.

(1) If I is right translation, then $(S \cap S^*)^{-1}(Q \cap \text{int}(Q^*) \cap \Psi(Q))$ is a nonempty intuitionistic fuzzy- I -open subset of $S^{-1}Q$.

(2) If I is left translation, then $(Q \cap \text{int}(Q^*) \cap \Psi(Q))(S \cap S^*)^{-1}$ is a nonempty intuitionistic fuzzy- I -open subset of QS^{-1} .

Proof. Let $U = V = X$ and apply Theorem 3.6. □

Theorem 3.8. Let (X, τ, I, \cdot) be an ifitg with identity e , $I \sim \tau$ and let $Q \in \mathcal{U}(X)$.

(1) If I is right translation, then $e \in \text{int}(Q^{-1}Q)$.

(2) If I is left translation, then $e \in \text{int}(QQ^{-1})$.

(3) If I is translation, then $e \in \text{int}(Q^{-1}Q) \cap \text{int}(QQ^{-1})$.

Proof. (1) We have $Q \cap \text{int}(Q^*) \cap \Psi(Q) \subseteq Q \cap Q^*$, such that from Theorem 3.7 (1), implies $e \in (Q \cap \text{int}(Q^*) \cap \Psi(Q))(Q \cap Q^*)^{-1}$.

The proofs of (2) and (3) are similar to (1). □

Proposition 3.9. *If $f : (X, \tau, I_1) \longrightarrow (Y, \varphi, I_2)$ is an intuitionistic fuzzy- I -homomorphism with $f(I_1) = I_2$. Then $f(\Psi(A)) = \Psi(f(A))$ for each $A \in I^X$.*

Theorem 3.10. *Let (X, τ, I, \cdot) be an ifitg with identity e , $I \cap \tau = 0_\sim$ and let $Q \in \mathcal{U}(X)$.*

- (1) *If I is right translation, then $e \in \text{int}(Q^{-1}Q)$.*
- (2) *If I is left translation, then $e \in \text{int}(QQ^{-1})$.*
- (3) *If I is translation, then $e \in \text{int}(Q^{-1}Q) \cap \text{int}(QQ^{-1})$.*

Proof. (2) Let $Q \in \mathcal{U}(X)$. Then there is $A \subseteq Q$ such that $A \in \beta(I, \tau)$. Since left translation by any element is an intuitionistic fuzzy- I -homomorphism, we have by Proposition 3.9, for an ifp $x_{(t,r)}$ in X , $x_{(t,r)}\Psi(A) = \Psi(x_{(t,r)}A)$. Then $x_{(t,r)}\Psi(A) \cap \Psi(A) = \Psi(x_{(t,r)}A \cap A)$. Thus if $x_{(t,r)}\Psi(A) \cap \Psi(A) \neq 0_\sim$, then $x_{(t,r)}A \cap A \neq 0_\sim$, since $\Psi(0_\sim) = 0_\sim$. Now we have

$$\begin{aligned} (\Psi(A))(\Psi(A))^{-1} &= \{x_{(t,r)} : x_{(t,r)}\Psi(A) \cap \Psi(A) \neq 0_\sim\} \\ &\subseteq \{x_{(t,r)} : x_{(t,r)}A \cap A \neq 0_\sim\} \\ &= AA^{-1} \subseteq QQ^{-1}. \end{aligned}$$

So $\Psi(A) \neq 0_\sim$. Since $\Psi(A)$ is intuitionistic fuzzy- I -open for any $A \in I^X$. Hence $e \in (\Psi(A))(\Psi(A))^{-1} \subseteq \text{int}(QQ^{-1})$.

- (1) The proof is similar to (2).
- (3) The proof follows from (1) and (2). □

Theorem 3.11. *Let (X, τ, I, \cdot) be an ifitg with $I \cap \tau = 0_\sim$ and let I be right or left translation. If Q is intuitionistic fuzzy subgroup and $Q \in \mathcal{U}(X)$. Then $Q = \text{int}(Q) = \text{Cl}(Q)$.*

Proof. By Theorem 3.10, $e \in Q^{-1}Q$ (or $e \in QQ^{-1}$). Then $Q = \text{int}(Q)$. □

Theorem 3.12. *Let (X, τ, I, \cdot) be an ifitg with $I \sim \tau$ and let I be right or left translation. If Q is intuitionistic fuzzy subgroup and $Q \in \mathcal{U}(X)$. Then $Q = \text{int}(Q) = \text{Cl}(Q)$.*

Proof. It is clear that $Q \in \mathcal{U}(X) \longrightarrow X \notin I$ and $I \sim \tau \longrightarrow \tau \cap I = 0_\sim$. Then by Theorem 3.10, $e \in Q^{-1}Q$ (or $e \in QQ^{-1}$). Thus $Q = \text{int}(Q)$. □

Definition 3.13. Let (X, τ, I, \cdot) be an ifitg and let $\beta(I, \tau)$ be a neighbourhood base of the identity e . Then for each ifp $x_{(t,r)}$ in X , the families

$$\beta_{x_{(t,r)}}(I, \tau) = \{x_{(t,r)}B : B \in \beta(I, \tau)\} \text{ and } \beta'_{x_{(t,r)}}(I, \tau) = \{Bx_{(t,r)} : B \in \beta(I, \tau)\}$$

are called both *neighbourhood bases* of $x_{(t,r)}$.

Theorem 3.14. *Let (X, τ, I, \cdot) be an ifitg and let $\beta(I, \tau)$ a neighbourhood base of the identity e . Then the following properties are satisfied:*

- (1) *for each $U, V \in \beta(I, \tau)$ there is $G \in \beta(I, \tau)$ such that $G \subseteq U \cap V$,*
- (2) *for each $U \in \beta(I, \tau)$ there is $V \in \beta(I, \tau)$ such that $VV \subseteq U$,*
- (3) *For every $U \in \beta(I, \tau)$ there is $V \in \beta(I, \tau)$ such that $V^{-1} \subseteq U$.*

Proof. (1) Every ifits (X, τ, I) satisfies this property.

(2) Let $U \in \beta(I, \tau)$. As γ is intuitionistic fuzzy continuous, $\gamma^{-1}(U)$ is a neighbourhood of e . Then there exist $V_1, V_2 \in \beta(I, \tau)$ such that $V_1 \times V_2 \subseteq \gamma^{-1}(U)$. By (1),

take $V \in \beta(I, \tau)$ such that $V \subseteq V_1 \cap V_2$. Then $V \times V \subseteq \gamma^{-1}(U)$. Thus by applying γ , we have that $VV \subseteq \gamma(\gamma^{-1}(U)) \subseteq U$.

(3) Let $U \in \beta(I, \tau)$. Since $\xi^{-1}(U)$ is a neighbourhood of e , there is $V \in \beta(I, \tau)$ such that $V \subseteq \xi^{-1}(U)$. By taking images by ξ , we have that $\xi(V) = V^{-1} \subseteq \xi(\xi^{-1}(U)) \subseteq U$. \square

Theorem 3.15. *Let (X, τ, I, \cdot) be an ifitg and let $\beta(I, \tau)$ a neighbourhood base of the identity e . Then for each fuzzy set A in X , $ICl(A) = \{AE : E \in \beta(I, \tau)\}$.*

Proof. Suppose $x_{(t,r)} \notin ICl(A)$. Then there is $E \in \beta(I, \tau)$ such that $x_{(t,r)} \notin AE$. Since $x_{(t,r)} \notin A$, by definition, there is an intuitionistic fuzzy- I -open neighbourhood G of e so that $x_{(t,r)}G \cap A = 0_{\sim}$. Let E satisfy the condition $E^{-1} \subseteq G$. Then $x_{(t,r)}E^{-1} \cap A = 0_{\sim}$, that is, $\{x_{(t,r)}\} \cap AE = 0_{\sim}$. Thus $x_{(t,r)} \notin AE$. \square

Definition 3.16. Let (X, τ, I, \cdot) be an ifitg. An ifs A in X is called *symmetric*, if $A = A^{-1}$.

Theorem 3.17. *Let (X, τ, I, \cdot) be an ifitg. Then each $l_{x_{(t,r)}} : X \rightarrow X(r_{x_{(t,r)}} : X \rightarrow X)$ is intuitionistic fuzzy- I -homomorphism. Where $l_{x_{(t,r)}}(y_{(t,r)}) = x_{(t,r)}y_{(t,r)}$ ($r_{x_{(t,r)}}(y_{(t,r)}) = y_{(t,r)}x_{(t,r)}$).*

Proof. Let $y_{(t,r)}$ be any ifp in X and let G be an intuitionistic fuzzy- I -open neighbourhood of $l_{x_{(t,r)}}(y_{(t,r)}) = x_{(t,r)}y_{(t,r)} = x_{(t,r)}(y_{(t,r)}^{-1})^{-1}$. Then there are intuitionistic fuzzy- I -open sets U and V containing $x_{(t,r)}$ and $y_{(t,r)}^{-1}$, respectively such that $UV^{-1} \subseteq G$. Especially, we have $x_{(t,r)}V^{-1} \subseteq G$. By Proposition 3.5 (5), the set V^{-1} is an intuitionistic fuzzy- I -open neighbourhood of $y_{(t,r)}$. Thus $l_{x_{(t,r)}}$ is intuitionistic fuzzy- I -continuous at $y_{(t,r)}$. Since $y_{(t,r)}$ is arbitrary, $l_{x_{(t,r)}}$ is intuitionistic fuzzy- I -continuous on X .

Now let A be an intuitionistic fuzzy- I -open set in X . Then by Proposition 3.5 (1), the set $l_{x_{(t,r)}}(A) = x_{(t,r)}A$ is intuitionistic fuzzy- I -open set in X . Thus $l_{x_{(t,r)}}$ is an intuitionistic fuzzy- I -open function. So $l_{x_{(t,r)}}$ is intuitionistic fuzzy- I -homomorphism.

In the same way, we proof $r_{x_{(t,r)}}$ is intuitionistic fuzzy- I -homomorphism. \square

Definition 3.18. An ifits (X, τ, I) is said to be an *intuitionistic fuzzy- I -homogeneous*, if for each ifp $x_{(t,r)}, y_{(t,r)}$ in X , there is an intuitionistic fuzzy- I -homomorphism f of the space X itself such that $f(x_{(t,r)}) = y_{(t,r)}$.

Theorem 3.19. *Every ifitg (X, τ, I, \cdot) is an intuitionistic fuzzy- I -homogeneous space.*

Proof. Let $x_{(t,r)}, y_{(t,r)}$ two ifp in X such that $z_{(t,r)} = x_{(t,r)}^{-1}y_{(t,r)}$. Then $l_{z_{(t,r)}}$ is an intuitionistic fuzzy- I -homomorphism of X and $l_{z_{(t,r)}}(x_{(t,r)}) = x_{(t,r)}(x_{(t,r)}^{-1}y_{(t,r)}) = y_{(t,r)}$. \square

Theorem 3.20. *Let (X, τ, I, \cdot) be an ifitg and let H be an intuitionistic fuzzy subgroup of X . If H contains a nonempty intuitionistic fuzzy- I -open set. Then H is intuitionistic fuzzy- I -open in X .*

Proof. Let $U \neq 0_{\sim}$ be any intuitionistic fuzzy- I -open set in X with $U \subseteq H$. Then for any $x_{(t,r)} \in H$, the set $l_{x_{(t,r)}}(U) = x_{(t,r)}U$ is intuitionistic fuzzy- I -open in X and is subset of H . Thus $H = \bigcup_{x_{(t,r)} \in H} (x_{(t,r)}U)$ is intuitionistic fuzzy- I -open in X . \square

Theorem 3.21. *Every intuitionistic fuzzy open subgroup H in an ifitg (X, τ, I, \cdot) is intuitionistic fuzzy ideal topological group and called intuitionistic fuzzy ideal topological subgroup of X .*

Proof. Let $x_{(t,r)}, y_{(t,r)}$ two ifp in H and let $G \in N(x_{(t,r)}y_{(t,r)}^{-1})$ relative to H there is intuitionistic fuzzy- I -open neighbourhoods $U \subseteq H$ of $x_{(t,r)}$ and $V \subseteq H$ of $y_{(t,r)}$ such that $UV^{-1} \subseteq G$. Since H is intuitionistic fuzzy open in X , G is an intuitionistic fuzzy open set in X . Since X is an intuitionistic fuzzy ideal topological group, there are intuitionistic fuzzy- I -open neighbourhoods W of $x_{(t,r)}$ and Q of $y_{(t,r)}$ such that $WQ^{-1} \subseteq G$. But the sets $U = W \cap H$ and $V = Q \cap H$ are intuitionistic fuzz- I -open sets in H , since H is fuzzy open. Also, $UV^{-1} \subseteq WQ^{-1} \subseteq G$. Then H is an intuitionistic fuzzy ideal topological group. \square

Lemma 3.22. *Let (X, τ, I, \cdot) be an ifitg and let H be intuitionistic fuzzy subgroup of X .*

- (1) $Cl(H)$ of H is intuitionistic fuzzy subgroup.
- (2) If H contains an intuitionistic fuzzy open set, then H is intuitionistic fuzzy- I -open.
- (3) If H is intuitionistic fuzzy open, then H is intuitionistic fuzzy- I -closed.
- (4) If H is intuitionistic fuzzy closed and of finite index in X , then H is intuitionistic fuzzy- I -open.

Theorem 3.23. *Let (X, τ, I, \cdot) be an ifitg and let A, B two ifp in X . Then*

- (1) $ICl(A)ICl(B) \subseteq Cl(AB)$,
- (2) $(ICl(A))^{-1} \subseteq Cl(A^{-1})$.

Proof. (1) Let $x_{(t,r)} \in ICl(A), y_{(t,r)} \in ICl(B)$ and let $G \in N(x_{(t,r)}y_{(t,r)})$. Then there is intuitionistic fuzzy- I -open neighbourhoods U and V such that $UV \subseteq G$. Since $x_{(t,r)} \in ICl(A), y_{(t,r)} \in ICl(B)$, there are $x_{(a_1, a_2)} \in A \cap U$ and $x_{(b_1, b_2)} \in B \cap V$. Thus $x_{(a_1, a_2)}x_{(b_1, b_2)} \in (A \cap U) \cap (B \cap V) \subseteq (AB) \cap G$. So $x_{(t,r)}y_{(t,r)} \in Cl(AB)$. Hence $ICl(A)ICl(B) \subseteq Cl(AB)$.

(2) Let $x_{(t,r)} \in (ICl(A))^{-1}$ and let $U \in N(x_{(t,r)})$. Since the inverse mapping is intuitionistic fuzz- I -open, $U^{-1} \in N(x_{(t,r)}^{-1})$. But $x_{(t,r)}^{-1} \in ICl(A), U^{-1} \cap A \neq 0_{\sim}$. Then $U \cap A^{-1} \neq 0_{\sim}$. Thus $x_{(t,r)} \in Cl(A^{-1})$. So $(ICl(A))^{-1} \subseteq Cl(A^{-1})$. \square

Theorem 3.24. *Let (X, τ, I, \cdot) be an ifitg. If $U \in N(e)$, then $U \subseteq ICl(U) \subseteq U^2$.*

Proof. Since $x_{(t,r)}U^{-1}$ is an intuitionistic fuzzy- I -open neighbourhood of $x_{(t,r)}$, there is $y_{(t,r)} \in U$ of form $x_{(t,r)}z_{(t,r)}^{-1}$ when $z_{(t,r)} \in U$. But $x_{(t,r)} = y_{(t,r)}z_{(t,r)} \in UU = U^2$. Then $U \subseteq ICl(U) \subseteq U^2$. \square

Theorem 3.25. *Let (X, τ, I, \cdot) be an ifitg. Then $ICl(A) \subseteq AU$ for every ifs A in X and every $U \in N(e)$.*

Proof. From Theorem 3.14 (3), for each $U \in N(e)$, there is $V \in N(e)$ such that $V^{-1} \subseteq U$. Let $x_{(t,r)} \in ICl(A)$ and $x_{(t,r)}V$ is an intuitionistic fuzzy- I -open neighbourhood of $x_{(t,r)}$. Then there is $z_{(t,r)} \in A \cap x_{(t,r)}V$, that is, $z_{(t,r)} \in x_{(t,r)}V$. Thus $z_{(t,r)} = z_{(t,r)}y_{(t,r)}^{-1} \in z_{(t,r)}V^{-1} \subseteq AU$. So $ICl(A) \subseteq AU$. \square

Theorem 3.26. *Let (X, τ, I, \cdot) be an ifitg. Then (X, τ, I) is intuitionistic fuzzy- I -regular and intuitionistic fuzzy ideal- T_2 space.*

Proof. Let F be an intuitionistic fuzzy closed in X and let $x_{(t,r)} \in F$. Multiply by $x_{(t,r)}^{-1}$ allows to assume that $x_{(t,r)} = e$. Since F is intuitionistic fuzzy closed, $G = X - F$ is an intuitionistic fuzzy open neighbourhood of e . Then there is intuitionistic fuzzy- I -open neighbourhood V of e such that $V^2 \subseteq G$. Thus $ICl(V) \subseteq G$. So $U = X - ICl(V)$ is an intuitionistic fuzzy- I -neighbourhood containing F which is disjoint from V . So (X, τ, I, \cdot) is intuitionistic fuzzy- I -regular. That is, $e \in V$ and $e \neq y_{(t,r)} \in F \subseteq U$ such that $V \cap U = 0_{\sim}$. Hence X is intuitionistic fuzzy ideal- T_2 -space. \square

Theorem 3.27. *Let (X, τ, I, \cdot) be an ifitg. If K is an intuitionistic fuzzy- I -compact in X , and F an intuitionistic fuzzy- I -closed set in X . Then FK and KF are intuitionistic fuzzy- I -closed sets in X .*

Proof. If $FK = X$, it is done. Let $x_{(t,r)} \in X - FK$. Then $F \cap x_{(t,r)}K^{-1} = 0_{\sim}$. Since K is intuitionistic fuzzy- I -compact in X , $x_{(t,r)}K^{-1}$ is intuitionistic fuzzy- I -compact. Thus there is an intuitionistic fuzzy- I -open neighbourhood U of e such that $F \cap Ux_{(t,r)}K^{-1} = 0_{\sim}$. So $FK \cap Ux_{(t,r)} = 0_{\sim}$. Since $Ux_{(t,r)}$ is intuitionistic fuzzy- I -open neighbourhood of $x_{(t,r)}$ contained in $X - FK$, FK is intuitionistic fuzzy- I -closed.

In same way to the proof of KF . \square

Theorem 3.28. *Let (X, τ, I, \cdot) be an ifitg and let H be an intuitionistic fuzzy subgroup in X . Then H is intuitionistic fuzzy- I -open set iff $Iint(H) \neq 0_{\sim}$.*

Proof. Let $x_{(t,r)} \in Iint(H)$. Then there is an intuitionistic fuzzy- I -open set U such that $x_{(t,r)} \in U \subseteq H$. For each $y_{(t,r)} \in H$, we have $y_{(t,r)}U \subseteq y_{(t,r)}H = H$. Since U is intuitionistic fuzzy- I -open, so is $y_{(t,r)}U$. Thus $H = \bigcup\{y_{(t,r)}U : y_{(t,r)} \in H\}$ is an intuitionistic fuzzy- I -open set.

The proof of the Converse is straightforward. \square

Theorem 3.29. *Let (X, τ, I, \cdot) be an ifitg and V be an intuitionistic fuzzy- I -open set in X . Then $A = \bigcup_{i=1}^n V^n$ is intuitionistic fuzzy- I -open set.*

Proof. Let V be an intuitionistic fuzzy- I -open set in X . Then by Proposition 3.5 (2), $VV = V^2$ is intuitionistic fuzzy- I -open set and $V^2V = V^3$ is intuitionistic fuzzy- I -open set, similarly, V^4, V^5, \dots . Thus the set $A = \bigcup_{i=1}^n V^n$ is intuitionistic fuzzy- I -open set. \square

Theorem 3.30. *Let (X, τ, I, \cdot) be an ifitg and let A be an ifs in X . Then $(Iint(A))^{-1} = Iint(A^{-1})$.*

Proof. Since $i : X \rightarrow X$ is intuitionistic fuzzy- I -homomorphism, $Iint(i(A)) = Iint(A^{-1}) = i(Iint(A)) = (Iint(A))^{-1}$. \square

Definition 3.31. Let (X, τ, I, \cdot) be an ifitg and let U be an intuitionistic fuzzy- I -open neighbourhood of identity e . An intuitionistic fuzzy set A in X is said to be U -fuzzy- I -disjoint, if $x_{(t,r)} \notin y_{(t,r)}U$ for any disjoint $x_{(t,r)}, y_{(t,r)} \in A$.

Definition 3.32. Let (X, τ, I, \cdot) be an ifitg. Then the family Ω of ifs in X is said to be intuitionistic fuzzy- I -discrete, provided for each ifp $x_{(t,r)}$ in X has an intuitionistic fuzzy- I -open neighbourhood that intersection one element of Ω .

Theorem 3.33. Let (X, τ, I, \cdot) be an ifitg and let U and V be an intuitionistic fuzzy- I -open neighbourhood of identity e such that $V^4 \subseteq U$ and $V^{-1} = V$. If an ifs A in X is intuitionistic fuzzy- I -disjoint, then the collection of intuitionistic fuzzy- I -open sets $\{z_{(t,r)}V : z_{(t,r)} \in A\}$ is intuitionistic fuzzy- I -discrete in X .

Proof. Enough verification, for each ifp $x_{(t,r)}$ in X , an intuitionistic fuzzy- I -open neighbourhood $x_{(t,r)}V$ of $x_{(t,r)}$ intersects at least one element of the collection $\{z_{(t,r)}V : z_{(t,r)} \in A\}$. Let suppose to the contrary that, for some ifp $x_{(t,r)}$ in X , there is distinct elements $z_{(t,r)}, y_{(t,r)} \in A$ such that $x_{(t,r)}V \cap z_{(t,r)}V \neq 0_{\sim}$ and $x_{(t,r)}V \cap y_{(t,r)}V \neq 0_{\sim}$. Then $x_{(t,r)}^{-1}z_{(t,r)} \in V^2$ and $y_{(t,r)}^{-1}x_{(t,r)} \in V^2$, where $y_{(t,r)}^{-1}z_{(t,r)} = (y_{(t,r)}^{-1}x_{(t,r)})(x_{(t,r)}^{-1}z_{(t,r)}) \in V^4$. Thus $z_{(t,r)} \in y_{(t,r)}U$. Contradiction with assumption that A is intuitionistic fuzzy- I -disjoint. \square

Theorem 3.34. Let (X, τ, I, \cdot) be an ifitg and let U be a symmetric intuitionistic fuzzy- I -open neighbourhood of identity e . Then $A = \bigcup_{i=1}^n U^n$ is intuitionistic fuzzy- I -open set and an intuitionistic fuzzy- I -closed fuzzy subgroup of X .

Proof. It is obvious from Lemma 3.22 and Theorem 3.29. \square

Theorem 3.35. Let (X, τ, I, \cdot) be an ifitg and let H be an intuitionistic fuzzy subgroup of X . Then H is intuitionistic fuzzy- I -discrete iff it has an intuitionistic fuzzy- I -isolated point.

Proof. Let $x_{(t,r)} \in H$ and $x_{(t,r)}$ is intuitionistic fuzzy- I -isolated in the relative topology of $H \subseteq X$. Then there is an intuitionistic fuzzy- I -open neighbourhood U of e in X such that $x_{(t,r)}U \cap H = \{x_{(t,r)}\}$. Thus for any $y_{(t,r)} \in H$, we have $y_{(t,r)}U \cap H = y_{(t,r)}U \cap \{y_{(t,r)}x_{(t,r)}^{-1}H\}$. So for every intuitionistic fuzzy point of H is intuitionistic fuzzy- I -isolated such that H is indeed intuitionistic fuzzy- I -discrete.

Conversely, suppose H is intuitionistic fuzzy- I -discrete. Then by definition, all of its intuitionistic fuzzy points are intuitionistic fuzzy- I -isolated. \square

Theorem 3.36. Let (X, τ, I, \cdot) be an ifitg and let U be an intuitionistic fuzzy neighbourhood of e . Then there is a symmetric intuitionistic fuzzy- I -open neighbourhood V of e such that $V \subseteq U$.

Proof. Let U be an intuitionistic fuzzy neighbourhood of e . Then there is an intuitionistic fuzzy open neighbourhood G of e such that $G \subseteq U$ and G^{-1} is intuitionistic fuzzy- I -open neighbourhood of e . Let $V = G \cap G^{-1} \neq 0_{\sim}$. Since V is intersection of intuitionistic fuzzy open and intuitionistic fuzzy- I -open sets, V is intuitionistic fuzzy- I -open and $V = V^{-1}$. \square

Theorem 3.37. Let (X, τ, I, \cdot) be an intuitionistic fuzzy ideal connected topological group and let H be an intuitionistic fuzzy subgroup of X . If H is an intuitionistic fuzzy- I -open, then $H=X$.

Proof. It is clear. \square

Definition 3.38. An iftg (X, τ, I, \cdot) with respect to intuitionistic fuzzy- I -continuity is an intuitionistic fuzzy group X endowed with an intuitionistic fuzzy topology such that for each ifp $x_{(t,r)}$ in X , the translations $l_{x_{(t,r)}}, r_{x_{(t,r)}} : X \rightarrow X, l_{x_{(t,r)}}(y_{(t,r)}) = x_{(t,r)}y_{(t,r)}, r_{x_{(t,r)}}(y_{(t,r)}) = y_{(t,r)}x_{(t,r)}$ are intuitionistic fuzzy- I -continuous, and such that the inverse mapping $i : X \rightarrow X, i(x_{(t,r)}) = x_{(t,r)}^{-1}$ is intuitionistic fuzzy- I -continuous.

Theorem 3.39. Let (X, τ, I, \cdot) be a Hausdorff iftg with respect to intuitionistic fuzzy- I -continuity such that left translations are intuitionistic fuzzy continuous (intuitionistic fuzzy- I -continuous), right translations are intuitionistic fuzzy- I -continuous (intuitionistic fuzzy continuous) and inverse mapping is intuitionistic fuzzy- I -continuous. For any ifs W in X , The intuitionistic fuzzy subgroup $C_X(W) = \{x_{(t,r)} \in X : y_{(t,r)}x_{(t,r)} = x_{(t,r)}y_{(t,r)}\}$ is intuitionistic fuzzy- I -closed in X .

Theorem 3.40. Let (X, τ, I, \cdot) be an intuitionistic fuzzy ideal connected topological group and let e be the identity. If U an intuitionistic fuzzy- I -open neighbourhood of e , then if X is generated by U .

Proof. Let U be an intuitionistic fuzzy- I -open neighbourhood of e . For every $n \in \mathbb{N}$, let U^n consisting of elements form $u_1 \dots u_n$, where $u_i \in U$. Let $G = \bigcup_{n=1}^{\infty} U^n$. Since every U^n is intuitionistic fuzzy- I -open, we have that G is an intuitionistic fuzzy- I -open set. Let X be an element of intuitionistic fuzzy- I -closure G , i.e., $x_{(t,r)} \in ICl(G)$. Since $x_{(t,r)}U^{-1}$ is an intuitionistic fuzzy- I -open neighborhood of $x_{(t,r)}$, it must intersect G . Then let $y_{(t,r)} \in G \cap x_{(t,r)}U^{-1}$. Since $y_{(t,r)} \in x_{(t,r)}U^{-1}$, then $y_{(t,r)} = x_{(t,r)}u^{-1}$ for some elements $u \in U$. Since $x_{(t,r)} \in G$, $x_{(t,r)} \in U^n$ for some $n \in \mathbb{N}$, i.e., $y_{(t,r)} = u_1 \dots u_n$ with every $u_i \in U$. Thus $x_{(t,r)} = u_1 \dots u_n u$, i.e., $x_{(t,r)} \in U^{n+1} \subseteq G$. So G is intuitionistic fuzzy- I -closed. Since X is intuitionistic fuzzy- I -connected and G is intuitionistic fuzzy- I -open and intuitionistic fuzzy- I -closed, we must have $G = X$. This means that X is generated by U . \square

4. CONCLUSIONS

This paper deals with intuitionistic fuzzy topological groups. One of the important subclasses is the class of groups. So, in the paper we studied the concept introduce a new class of topological groups called intuitionistic fuzzy ideal topological groups by depended on an intuitionistic fuzzy topological groups (X, τ) .

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