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Abstract. In this paper, we introduce the notion of quasi-interior ideal and fuzzy soft quasi-interior ideal of Γ-semiring and we characterize the regular Γ-semiring in terms of fuzzy soft quasi-interior ideal of Γ-semiring.

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Keywords: Semiring, Γ-semiring, Γ-regular semiring, Bi-quasi ideal, Bi-interior ideal, Fuzzy bi-quasi ideal, Fuzzy bi-interior ideal, Quasi-interior ideal and fuzzy soft quasi-interior ideal.

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1. Introduction

Semiring is the best algebraic structure which is a common generalization of rings and distributive lattices, was first introduced by Vandiver [1] in 1934 but non-trivial examples of semirings had appeared in the studies on the theory of commutative ideals of rings by Dedekind in 19th century. Semiring is an universal algebra with two binary operations called addition and multiplication, where one of them distributive over the other. A natural example of semiring is the set of all natural numbers under usual addition and multiplication of numbers. In particular, if I is the unit interval on the real line, then $(I, \max, \min)$ is a semiring in which 0 is the additive identity and 1 is the mutiplicative identity. The theory of rings and the theory of semigroups have considerable impact on the development of the theory of semirings. In structure, semirings lie between semigroups and rings. Additive and multiplicative structures of a semiring play an important role in determining the structure of a semiring. Semiring as the basic algebraic structure was used in the areas of theoretical computer science as well as in the solutions of graph theory, optimization theory and in particular for studying automata, coding theory and formal languages. Semiring theory has many applications in other branches. In 1995, Murali Krishna Rao [2, 3, 4] introduced the notion of a Γ-semiring as a generalization of a Γ-ring, ring, ternary semiring and semiring. In structure, semirings lie between semigroups and rings. Historically semirings first appear implicitly in Dedekind and later in Macaulay, Neither and Lorenzen in connection with the study of a ring. However semirings first appear explicitly in Vandiver, also in connection with the axiomatization of Arithmetic of natural numbers. Semirings have been studied by various researchers in an attempt to broaden techniques coming from semigroup theory, ring theory or in connection with applications. The developments of the theory in semirings have been taking place since 1950. The results which hold in rings but not in semigroups hold in semirings, since semiring is a generalization of a ring. The study of rings shows that multiplicative structure of a ring is independent of additive structure whereas in a semiring multiplicative structure depends on additive structure. The theory of rings and the theory of semigroups have considerable impact on the development of the theory of semirings. Semirings play an important role in studying matrices and determinants. Semirings are useful in the areas of theoretical computer science as well as in the solution of graph theory, optimization theory, in particular for studying automata, coding theory and formal languages. Semiring theory has many applications in other branches of mathematics.

We know that the notion of a one sided ideal of any algebraic structure is a generalization of the notion of an ideal. The quasi ideals are generalization of left ideals and right ideals whereas the bi-ideals are generalization of quasi ideals. In 1952, the concept of bi-ideals was introduced by Good and Hughes [5] for semigroups. The notion of bi-ideals in rings and semirings were introduced by Lajos and Szasz [6, 7]. In 1956, Steinfeld [8] first introduced the notion of quasi ideals for semigroups and then for rings. Iseki [9, 10, 11] introduced the concept of quasi ideal for a semiring. Henriksen [12] studied ideals in semirings. Quasi ideals in Γ-semirings studied by Jagtap and Pawar [13]. Rao [14, 15, 16, 17, 18, 19, 20] introduced and studied bi-quasi-ideals in semirings, bi-quasi-ideals and fuzzy bi-quasi ideals in Γ-semigroups. In this paper, as a further generalization of ideals, the notion of quasi-interior ideal of Γ-semiring, as a generalization of ideal, left ideal, right ideal, bi-ideal, quasi ideal and interior ideal of Γ-semiring. Further the notion of fuzzy quasi-interior ideal of Γ-semiring and we characterize the regular Γ-semiring in terms of fuzzy quasi-interior ideal of Γ-semiring and studied some of their properties.

The fuzzy set theory was developed by Zadeh [21] in 1965. Many papers on fuzzy sets appeared showing the importance of the concept and its applications to logic, set theory, group theory, ring theory, real analysis, topology, measure theory etc. The fuzzification of algebraic structure was introduced by Rosenfeld and he introduced the notion of fuzzy subgroups in 1971. Swamy and Swamy [22]studied fuzzy prime ideals in rings in 1988. In 1982, Liu [23] defined and studied fuzzy subrings as well as fuzzy ideals in rings. Mandal [24] studied fuzzy ideals and fuzzy interior ideals in an ordered semiring. Rao [25] studied fuzzy soft Γ-semiring and fuzzy soft k-ideal over Γ-semiring. Jun et al. [26] and Muhiudin et al. [27, 28, 29] studied soft sets and ideals overs soft serts in ordered semigroups. Kuroki [30] studied fuzzy interior ideals in semigroups. Rao [17] studied T-fuzzy ideals of ordered Γ-semirings.

In this paper, we introduce the notion of quasi-interior ideal and fuzzy soft quasi-interior ideal of Γ-semiring and we characterize the regular $\Gamma-$semiring in terms of fuzzy soft quasi-interior ideal of Γ-semiring.

2. Preliminaries

In this section, we recall some of the fundamental concepts and definitions which are necessary for this paper.

Definition 2.1 ([14]). A set S together with two associative binary operations called addition and multiplication (denoted by $+$ and $\cdot$ respectively) will be called *semiring*, provided that

(i) addition is a commutative operation,
(ii) multiplication distributes over addition both from the left and from the right,
(iii) there exists $0 \in S$ such that $x + 0 = x$ and $x \cdot 0 = 0 \cdot x = 0$ for all $x \in S$.

Definition 2.2 ([14]). Let M and Γ be two non-empty sets. Then M is called a Γ-*semigroup*, if it satisfies the following:

(i) $x\alpha y \in M$,
(ii) $x\alpha(y\beta z) = (x\alpha y)\beta z$ for all $x, y, z \in M,\ \alpha, \beta \in \Gamma$.

Definition 2.3 ([14]). Let $(M, +)$ and $(\Gamma, +)$ be commutative semigroups. A Γ-semigroup M is said to be Γ-*semiring*, if it satisfies the following axioms: for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$,

(i) $x\alpha(y + z) = x\alpha y + x\alpha z$,
(ii) $(x + y)\alpha z = x\alpha z + y\alpha z$,
(iii) $x(\alpha + \beta)y = x\alpha y + x\beta y$.

Every semiring M is a Γ-semiring with $\Gamma = M$ and ternary operation as the usual semiring multiplication

Definition 2.4 ([14]). A Γ-semiring M is said to *have zero element*, if there exists an element $0 \in M$ such that $0 + x = x = x + 0$ and $0\alpha x = x\alpha 0 = 0$, for all $x \in M$.

Example 2.5. Let M be the additive semi group of all $m \times n$ matrices over the set of non negative rational numbers and Γ be the additive semigroup of all $n \times m$ matrices over the set of non negative integers, then with respect to usual matrix multiplication M is a Γ-semiring.

Definition 2.6 ([15]). Let M be a Γ-semiring and A be a non-empty subset of M. A is called a Γ-*subsemiring* of M, if A is a sub-semigroup of $(M, +)$ and $A\Gamma A \subseteq A$.

Definition 2.7 ([15]). Let M be a Γ-semiring. A subset A of M is called a *left* (*right*) *ideal* of M, if A is closed under addition and $M\Gamma A \subseteq A$ ($A\Gamma M \subseteq A$). A is called an *ideal* of M, if it is both a left ideal and a right ideal of M .

Definition 2.8 ([14]). Let M be a Γ-semiring. An element $a \in M$ is said to be a *regular element* of M, if there exist $x \in M, \alpha, \beta \in \Gamma$ such that $a = a\alpha x\beta a$.

Definition 2.9 ([14]). Let M be a Γ-semiring. If every element of M is a regular, then M is said to be a *regular* Γ-*semiring*.

Definition 2.10 ([14]). An element $a \in M$ is said to be *idempotent* of M if $a = a\alpha a$ for all $\alpha \in \Gamma$.

Definition 2.11 ([15]). Every element of M is an idempotent of M then M is said to be an *idempotent* Γ-*semiring* M.

Definition 2.12 ([19]). A non-empty subset A of Γ-semiring M is called:

(i) a Γ-*subsemiring* of M, if $(A,+)$ is a subsemigroup of $(M,+)$ and $A\Gamma A \subseteq A$,
(ii) a *quasi ideal* of M, if A is a Γ-subsemiring of M and $A\Gamma M \cap M\Gamma A \subseteq A$,
(iii) a *bi-ideal* of M, if A is a Γ-subsemiring of M and $A\Gamma M\Gamma A \subseteq A$,
(iv) an *interior ideal* of M, if A is a Γ-subsemiring of M and $M\Gamma A\Gamma M \subseteq A$,
(v) a $left$ $(right)$ *ideal* of M, if A is a Γ-subsemiring of M and $M\Gamma A \subseteq A(A\Gamma M \subseteq A)$,
(vi) an *ideal*, if A is a Γ-subsemiring of $M, A\Gamma M \subseteq A$ and $M\Gamma A \subseteq A$,
(vii) a k-*ideal*, if A is a Γ-subsemiring of $M, A\Gamma M \subseteq A, M\Gamma A \subseteq A$ and $x \in M,\ x+y \in A, y \in A$ imply $x \in A$,
(viii) a $left(right)$ *bi-quasi ideal* of M, if A is a Γ-subsemiring of M and $A\Gamma M \cap M\Gamma M\Gamma A(A\Gamma M \cap M\Gamma A\Gamma M) \subseteq A$,
(ix) a *bi- quasi ideal* of M, if A is a left bi- quasi ideal and a right bi- quasi ideal of M.

Definition 2.13 ([19]). Let M be a non-empty set. A mapping $f : M \to [0,1]$ is called a *fuzzy subset* of Γ-semiring M. If f is not a constant function then f is called a *non-empty fuzzy subset*.

Definition 2.14 ([19]). Let f be a fuzzy subset of a non-empty set M. Then for $t \in [0,1]$, the set $f_t = \{x \in M \mid f(x) \geq t\}$ is called a *level subset* of M with respect to f.

Definition 2.15 ([19]). Let M be a Γ-semiring. A fuzzy subset μ of M is called a *fuzzy* Γ-*subsemiring* of M, if it satisfies the following conditions:
(i) $\mu(x+y) \geq min\ \{\mu(x), \mu(y)\}$,
(ii) $\mu(x\alpha y) \geq min\ \{\mu(x), \mu(y)\}$ for all $x, y \in M, \alpha \in \Gamma$.

Definition 2.16 ([19]). A fuzzy subset μ of Γ-semiring M is called a *fuzzy left* $(right)$ *ideal* of M, if it satisfies the following conditions: for all $x, y \in M, \alpha \in \Gamma$,
(i) $\mu(x+y) \geq min\{\mu(x), \mu(y)\}$,
(ii) $\mu(x\alpha y) \geq \mu(y)\ (\mu(x))$.

Definition 2.17 ([19]). A fuzzy subset μ of Γ-semiring M is called a *fuzzy ideal* of M, if it satisfies the following conditions: for all $x, y \in M, \alpha \in \Gamma$,
(i) $\mu(x+y) \geq min\{\mu(x), \mu(y)\}$,
(ii) $\mu(x\alpha y) \geq max\ \{\mu(x), \mu(y)\}$.

Definition 2.18. For any two fuzzy subsets λ and μ of M, $\lambda \subseteq \mu$ means $\lambda(x) \leq \mu(x)$ for all $x \in M$.

Definition 2.19 ([19]). Let f and g be fuzzy subsets of Γ-semiring M. Then $f \circ g, f+g, f \cup g, f \cap g$, are defined by: for all $z \in M$,

$$f \circ g(z) = \begin{cases} \sup\limits_{z=x\alpha y} \{min\{f(x), g(y)\}\}, \\ 0, \quad \text{otherwise.} \end{cases} ;\ f+g(z) = \begin{cases} \sup\limits_{z=x+y} \{min\{f(x), g(y)\}\}, \\ 0, \quad \text{otherwise} \end{cases}$$

$$f \cup g(z) = max\{f(z), g(z)\} \ \ ;\ f \cap g(z) = min\{f(z), g(z)\},$$

where $x, y \in M, \alpha \in \Gamma$.

Definition 2.20 ([19])**.** A function $f : R \to M$, where R and M are Γ-semirings. Then f is called a Γ-semiring homomorphism, if $f(a+b) = f(a)+f(b)$ and $f(a\alpha b) = f(a)\alpha f(b)$ for all $a, b \in R, \alpha \in \Gamma$.

Definition 2.21 ([19])**.** Let A be a non-empty subset of M. The *characteristic function* of A is a fuzzy subset of M, defined by

$$\chi_{_A}(x) = \begin{cases} 1, & \text{if } x \in A; \\ 0, & \text{if } x \notin A. \end{cases}$$

3. Quasi-interior ideals of Γ-semirings

In this section, we introduce the notion of quasi-interior ideal as a generalization of quasi-ideal and interior ideal of Γ-semiring and study the properties of quasi-interior ideal of Γ-semiring. Throughout this paper, M is a Γ-semiring with unity element.

Definition 3.1. A non-empty subset B of M is called a *left quasi-interior ideal* of M, if B is a Γ-subsemiring of M and $M\Gamma B\Gamma M\Gamma B \subseteq B$.

Definition 3.2. A non-empty subset B of M is called a *right quasi-interior ideal* of M, if B is a Γ-subsemiring of M and $B\Gamma M\Gamma B\Gamma M \subseteq B$.

Definition 3.3. A non-empty subset B of M is called a *quasi-interior ideal* of M, if B is a Γ-subsemiring of M and $B$is left and right quasi-interior ideal of M.

Remark 3.4. A quasi-interior ideal of a Γ-semiring M need not be quasi-ideal, interior ideal, bi-interior ideal. and bi-quasi ideal of Γ-semiring M.

Example 3.5. Let Q be the set of all rational numbers and let

$$M = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \mid a, b, c \in Q \right\}.$$

Then M is a Γ-semiring with respect to usual addition of matrices and ternary operation is defined as the usual matrix multiplication. If $A = \left\{ \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \mid a, 0 \neq b \in Q \right\}$, then A is a right quasi-interior ideal but not a bi-ideal of M.

In the following theorem, we mention some important properties and we omit the proofs since they are straight forward.

Theorem 3.6. *Let M be a Γ-semring. Then the following are hold.*

1. *Every left ideal is a left quasi-interior ideal of M.*
2. *Every right ideal is a right quasi-interior ideal of M.*
3. *Every quasi ideal is a quasi-interior ideal of M.*
4. *Every ideal is a quasi-interior ideal of M.*
5. *Intersection of a right ideal and a left ideal of M is a quasi-interior ideal of M.*
6. *If L is a left ideal and R is a right ideal of M then $B = R\Gamma L$ is a quasi-interior ideal of M.*
7. *If B is a quasi-interior ideal and T is a Γ-subsemiring of M, then $B \cap T$ is a quasi-interior ideal of ring M.*

(8) *Let B be a Γ-subsemiring of M. If $M\Gamma M\Gamma M\Gamma B \subseteq B$, then B is a left quasi-interior ideal of M.*
(9) *Let B be a Γ-subsemiring of M. If $M\Gamma M\Gamma M\Gamma B \subseteq B$ and $B\Gamma M\Gamma M\Gamma M \subseteq B$, then B is a quasi-interior ideal of M.*
(10) *Intersection of a right quasi-interior ideal and a left quasi-interior ideal of M is a quasi-interior ideal of M.*
(11) *If L is a left ideal and R is a right ideal of M, then $B = R \cap L$ is a quasi-interior ideal of M.*

Theorem 3.7. *If B be a left quasi-interior ideal of a Γ-semiring M, then B is a left bi-quasi ideal of M.*

Proof. Suppose B is a left quasi-interior ideal of M. Then $M\Gamma B\Gamma M\Gamma B \subseteq B$.Thus we have $B\Gamma M\Gamma B \subseteq M\Gamma B\Gamma M\Gamma B$. So we get

$$M\Gamma B \cap B\Gamma M\Gamma B \subseteq B\Gamma M\Gamma B \subseteq M\Gamma B\Gamma M\Gamma B \subseteq B.$$

Hence B is a left bi-quasi ideal of M. □

Corollary 3.8. *If B be a right quasi-interior ideal of a Γ-semiring M, then B is a right bi-quasi ideal of M.*

Corollary 3.9. *If B be a quasi-interior ideal of a Γ-semiring M, then B is a bi-quasi ideal of M.*

Theorem 3.10. *If B be a left quasi-interior ideal Γ-semiring of M, then B is a bi-interior ideal of M.*

Proof. Suppose B is a left quasi-interior ideal of M. Then we have

$$M\Gamma B\Gamma M\Gamma B \subseteq B.$$

Thus we get

$$M\Gamma B\Gamma M \cap B\Gamma M\Gamma B \subseteq B\Gamma M\Gamma B \subseteq M\Gamma B\Gamma M\Gamma B \subseteq B.$$

So B is a bi-interior ideal of M. □

Corollary 3.11. *If B be a right quasi-interior ideal of a Γ-semiring M, then B is a bi-interior ideal of M.*

Corollary 3.12. *If B be a quasi-interior ideal of a Γ-semiring M, then B is a bi-interior ideal of M.*

Theorem 3.13. *Every left quasi-interior ideal of a Γ-semiring M is a bi-ideal of M.*

Proof. Let B be a left quasi-interior ideal of M. Then we have

$$B\Gamma M\Gamma B \subseteq M\Gamma B\Gamma M\Gamma B \subseteq B.$$

Thus $B\Gamma M\Gamma B \subseteq B$. So B is a bi-ideal of M. □

Corollary 3.14. *Every right quasi-interior ideal of a Γ-semiring M is a bi-ideal of M.*

Corollary 3.15. *Every quasi-interior ideal of a Γ-semiring M is a bi-ideal of M.*

4. Fuzzy soft quasi interior ideals of Γ-semiring M

In this section, we introduce the notion of fuzzy soft right(left) quasi-interior ideals and fuzzy soft interior ideal and study the properties of fuzzy soft quasi ideals.

Definition 4.1. Let U be an initial Universe set and E be the set of parameters. Let $P(U)$ denotes the power set of U. A pair (f, E) is called a *soft set over* U, where f is a mapping given by $f : E \to P(U)$.

Definition 4.2. For a soft set (f, A), the set $\{x \in A \mid f(x) \neq \varnothing\}$ is called the *support* of (f, A) denoted by $Supp(f, A)$. If $Supp(f, A) \neq \varnothing$, then (f, A) is called a *non null soft set*.

Definition 4.3. Let U be an initial Universe set and E be the set of parameters. Let $A \subseteq E$. A pair (f, A) is called a *fuzzy soft set over* U, where $f : A \to I^U$ and I^U denotes the collection of all fuzzy subsets of U.

Definition 4.4. Let $(f, A), (g, B)$ be fuzzy soft sets over U. Then (f, A) is said to be a *fuzzy soft subset* of (g, B), denoted by $(f, A) \subseteq (g, B)$, if $A \subseteq B$ and $f(a) \subseteq g(a)$ for all $a \in A$.

Definition 4.5. Let (f, A), (g, B) be fuzzy soft sets. The *intersection* of (f, A) and (g, B), denoted by $(f, A) \cap (g, B) = (h, C)$, where $C = A \cup B$, is defined as:

$$h_c = \begin{cases} f_c, & \text{if } c \in A \setminus B; \\ g_c, & \text{if } c \in B \setminus A; \\ f_c \cap g_c, & \text{if } c \in A \cap B. \end{cases}$$

Definition 4.6. Let (f, A), (g, B) be fuzzy soft sets over U. "(f, A) and (g, B)", denoted by "$(f, A) \wedge (g, B)$" is defined by $(f, A) \wedge (g, B) = (h, C)$, where $C = A \times B$. $h_c(x) = min \ \{f_a(x), g_b(x)\}$ for all $c = (a, b) \in A \times B$ and $x \in U$.

Definition 4.7. Let S be a Γ-semiring and E be a parameter set and $A \subseteq E$. Let $f : A \to [0, 1]^S$ be a mapping, where $[0, 1]^S$ denotes the collection of all fuzzy subsets of S. Then (f, A) is called a *fuzzy soft left* (*right*) *ideal over* S, if for each $a \in A$, the corresponding fuzzy subset $f_a : S \to [0, 1]$ is a fuzzy left(right) ideal of S, i.e., for all $x, y \in S, \alpha, \beta \in \Gamma$,

(i) $f_a(x + y) \geq \min \{f_a(x), f_a(y)\}$,
(ii) $f_a(x\alpha y) \geq f_a(y)(f_a(x))$.

Definition 4.8. Let S be a Γ-semiring, let E be a parameter set, let $A \subseteq E$ and let $f : A \to [0, 1]^S$ be a mapping. Then (f, A) is called a *fuzzy soft ideal* over S, if for each $a \in A$, the corresponding fuzzy subset $f_a : S \to [0, 1]$ is a fuzzy ideal of S, i.e., for all $x, y \in S, \alpha, \beta \in \Gamma$,

(i) $f_a(x + y) \geq \min \{f_a(x), f_a(y)\}$,
(ii) $f_a(x\alpha y) \geq \max \{f_a(x), f_a(y)\}$.

Definition 4.9. Let S be a Γ-semiring, let E be a parameter set, $A \subseteq E$ and let $f : A \to [0, 1]^S$ be a mapping. Then (f, A) is called a *fuzzy soft interor ideal* over S, if for each $a \in A$, the corresponding fuzzy subset $f_a : S \to [0, 1]$ is a fuzzy interior ideal of S, i.e., for all $x, y \in S, \alpha, \beta \in \Gamma$,

(i) $f_a(x + y) \geq \min \{f_a(x), f_a(y)\}$,
(ii) $f_a(x\alpha y\beta z) \geq \max \{f_a(y)\}$.

Definition 4.10. Let S be a Γ-semiring, E be a parameter set, let $A \subseteq E$ and let $f : A \to [0,1]^S$ be a mapping. Then (f, A) is called a *fuzzy soft quasi ideal* over S, if for each $a \in A$, the corresponding fuzzy subset $f_a : S \to [0,1]$ is a fuzzy quasi ideal of S, i.e. for all $x, y \in S$,

(i) $f_a(x+y) \geq \min(f_a(x), f_a(y))$,

(ii) $f_a \circ \chi_S \wedge \chi_S \circ f_a \subseteq f_a$.

Definition 4.11. Let $(f, A), (g, B)$ be fuzzy soft ideals over a Γ-semiring S. The *product* of (f, A) and (g, B), denoted by $((f \circ g), C)$, where $C = A \cup B$, is defined as: for all $c \in A \cup B$ and $x \in S, \alpha \in \Gamma$,

$$(f \circ g)_c(x) = \begin{cases} f_c(x), & \text{if } c \in A \setminus B; \\ g_c(x), & \text{if } c \in B \setminus A; \\ \underset{x=a\alpha b}{Sup}\{min\{f_c(a), g_c(b)\}\}, & \text{if } c \in A \cap B. \end{cases}$$

Definition 4.12. Let M be Γ-semiring, E be a parameter set, $A \subseteq E$ and let $\mu : A \to [0,1]^S$ be a mapping. Then (μ, A) is called a *fuzzy soft left (right) quasi-interior ideal* over M, if for each $a \in A$, and for all $x, y \in M$,

(i) $\mu_a(x+y) \geq \min\{\mu_a(x), \mu_a(y)\}$,

(ii) $\chi_M \circ \mu_a \circ \chi_M \circ \mu_a \subseteq \mu_a \big(\mu_a \circ \chi_M \circ \mu_a \circ \chi_M \subseteq \mu_a\big)$.

A fuzzy soft set (μ, A) of Γ-semiring M is called a *fuzzy soft quasi-interior ideal*, if it is both fuzzy soft left and right quasi-interior ideal of M.

Example 4.13. Let Q be the set of all rational numbers and let

$$M = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \mid a, b, c \in Q \right\}.$$

Then M is a Γ-semiring with respect to usual addition of matrices and ternary operation is defined as the usual matrix multiplication. If $A = \left\{ \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \mid a, 0 \neq b \in Q \right\}$, then A is a right quasi-interior ideal but not a bi-ideal of Γ-semiring M. Define $\mu : M \to [0,1]$ such that $\mu(x) == \begin{cases} 1 & \text{if } x \in A, \\ 0, & \text{otherwise.} \end{cases}$

Then μ is a fuzzy right quasi-interior ideal of M.

Theorem 4.14. *Let M be a Γ-semiring, let E be a parameter set and let $A \subseteq E$. If (μ, A) is a fuzzy soft right ideal over M, then (μ, A) is a fuzzy soft right quasi-interior ideal over M.*

Proof. Let (μ, A) be a fuzzy soft right ideal over M. Then for each $a \in A$, μ_a is a fuzzy soft right ideal of the Γ-semiring. Let $x \in M$. Then we have

$$\begin{aligned} \mu_a \circ \chi_M(x) &= \sup_{x=b\alpha c} \min\{\mu_a(b), \chi_M(c)\} b, c \in M, \alpha \in \Gamma \\ &= \sup_{x=b\alpha c} \mu_a(b) \\ &\leq \sup_{x=b\alpha c} \mu_a(b\alpha c) \\ &= \mu_a(x). \end{aligned}$$

Thus $\mu_a \circ \chi_M(x) \leq \mu_a(x)$. Now we get

$$\mu_a \circ \chi_M \circ \mu_a \circ \chi_M(x) = \sup_{x=u\alpha v\beta s} \min\{\mu_a \circ \chi_M(u\alpha v), \mu_a \circ \chi_M(s)\}$$

$$\leq \sup_{x=u\alpha v\beta s} \min\{\mu(u\alpha v), \mu(s)\}$$
$$= \mu_a(x).$$

So (μ, A) is a fuzzy soft right quasi-interior ideal of M. □

Corollary 4.15. *Every fuzzy soft left ideal of a Γ-semiring M is a fuzzy soft left quasi-interior ideal of M.*

Corollary 4.16. *Every fuzzy soft ideal of a Γ-semiring M is a fuzzy soft quasi-interior ideal of M.*

Theorem 4.17. *Let M be a Γ-semiring and let μ be a non-empty fuzzy subset of M. Then μ is a fuzzy left quasi-interior ideal of M if and only if the level subset μ_t of μ is a left quasi-interior ideal of M for every $t \in [0, 1]$, where $\mu_t \neq \phi$.*

Proof. Let M be a Γ-semiring and μ be a non-empty fuzzy subset of M. Suppose μ is a fuzzy left quasi-interior ideal of a $\Gamma-$semiring M, $\mu_t \neq \phi, t \in [0, 1]$ and $a, b \in \mu_t$. Then $\mu(a) \geq t, \mu(b) \geq t$. Thus $\mu(a + b) \geq \min\{\mu(a), \mu(b)\} \geq t$. So $a + b \in \mu_t$.

Now let $x \in M\Gamma\mu_t\Gamma M\Gamma\mu_t$. Then $x = b\alpha a\beta d\gamma c$, where $b, d \in M, a, c \in \mu_t, \alpha, \beta and\gamma \in \Gamma$. Thus $\chi_M \circ \mu \circ \chi_M \circ \mu(x) \geq t$. So $\mu(x) \geq \chi_M \circ \mu \circ \chi_M \circ \mu(x) \geq t$. Hence $x \in \mu_t$. Therefore μ_t is a left quasi-interior ideal of$M$.

Conversely, suppose that μ_t is a left quasi-interior ideal of the $\Gamma-$ semiring M for all $t \in Im(\mu)$. Let $x, y \in M, \alpha \in \Gamma, \mu(x) = t_1, \mu(y) = t_2$ and $t_1 \geq t_2$ and let $x, y \in \mu_{t_2}$. Then $x+y \in \mu_{t_2}$ and $x\alpha y \in \mu_{t_2}$. Thus $\mu(x+y) \geq t_2 = \min\{t_1, t_2\} = \min\{\mu(x), \mu(y)\}$. So $\mu(x + y) \geq t_2 = \min\{\mu(x), \mu(y)\}$. Moreover, We have $M\Gamma\mu_l\Gamma M\Gamma\mu_l \subseteq \mu_t$ for all $l \in Im(\mu)$. Let $t = \min\{Im(\mu)\}$. Then $M\Gamma\mu_t M\Gamma\mu_t\Gamma \subseteq \mu_t$. Thus $\chi_M \circ \mu \circ \chi_M \circ \mu \subseteq \mu$. So μ is a fuzzy left quasi-interior ideal of M. □

Corollary 4.18. *Let M be a Γ-semiring and let μ be a non-empty fuzzy subset of M. Then μ is a fuzzy right quasi-interior ideal of a Γ-semiring if and only if the level subset μ_t of μ is a right quasi-interior ideal of M for every $t \in [0, 1]$, where $\mu_t \neq \phi$.*

Theorem 4.19. *Let M be a Γ-semiring, let E be a parameter set and let $A \subseteq E$. Then (I, A) is a soft right quasi-interior ideal of M if and only if (χ_I, A) is a fuzzy soft right quasi-interior ideal of M.*

Proof. Suppose (I, A) is a soft right quasi-interior ideal of M. Then for each $a \in A$, I_a is a right quasi-interior ideal of M.

Obviously, χ_I is a fuzzy Γ-subsemiring of M. We have $I_a\Gamma M\Gamma I_a\Gamma M \subseteq I_a$. Then

$$\chi_I \circ \chi_M \circ \chi_I \circ \chi_M = \chi_{I_a\Gamma M\Gamma I_a\Gamma M}$$
$$= \chi_{I_a\Gamma M\Gamma I_a\Gamma M}$$
$$\subseteq \chi_I.$$

Thus χ_I is a fuzzy right quasi-interior ideal of$M$. So (χ_I, A) is a fuzzy soft quasi-interior ideal of$M$.

Conversely, suppose that (χ_I, A) is a fuzzy soft quasi-interior ideal of M. Then χ_I is a fuzzy right quasi-interior ideal of M. Since I is a $\Gamma-$subsemiring of M,

$$\chi_I \circ \chi_M \circ \chi_I \circ \chi_M \subseteq \chi_I.$$

Thus $\chi_{I_a\Gamma M\Gamma I_a\Gamma M} \subseteq \{\chi_I\}_a$. So $I_a\Gamma M\Gamma I_a\Gamma M \subseteq I_a$. Hence I_a is a right quasi-interior ideal of M. Therefore (I, A) is a fuzzy soft right quasi-interior ideal of M. □

Theorem 4.20. *If μ and λ are fuzzy left quasi-interior ideals of M, then $\mu\cap\lambda$ is a fuzzy left quasi-interior ideal of M.*

Proof. Let μ and λ be fuzzy left quasi-interior ideals of M, let $x, y \in M$ and let $\alpha, \beta \in \Gamma$. Then

$$\begin{aligned}
\mu\cap\lambda(x+y) &= \min\{\mu(x+y), \lambda(x+y)\}\\
&\geq \min\{\min\{\mu(x), \mu(y)\}, \min\{\lambda(x), \lambda(y)\}\}\\
&= \min\{\min\{\mu(x), \lambda(x)\}, \min\{\mu(y), \lambda(y)\}\}\\
&= \min\{\mu\cap\lambda(x), \mu\cap\lambda(y)\}\\
\chi_M\circ\mu\cap\lambda(x) &= \sup_{x=a\alpha b} \min\{\chi_M(a), \mu\cap\lambda(b)\}\\
&= \sup_{x=a\alpha b} \min\{\chi_M(a), \min\{\mu(b), \lambda(b)\}\}\\
&= \sup_{x=a\alpha b} \min\{\min\{\chi_M(a), \mu(b)\}, \min\{\chi_M(a), \lambda(b)\}\}\\
&= \min\{\sup_{x=a\alpha b} \min\{\chi_M(a), \mu(b)\}, \sup_{x=a\alpha b} \min\{\chi_M(a), \lambda(b)\}\}\\
&= \min\{\chi_M\circ\mu(x).\chi_M\circ\lambda(x)\}\\
&= \chi_M\circ\mu\cap\chi_M\circ\lambda(x).
\end{aligned}$$

Thus we have

$$\begin{aligned}
&\chi_M\circ\mu\cap\chi_M\circ\lambda\\
&= \chi_M\circ\mu\cap\chi_M\circ\lambda\\
&= \chi_M\circ\mu\cap\lambda\circ\chi_M\circ\mu\cap\lambda(x)\\
&= \sup_{x=a\alpha b\beta c} \min\{\chi_M\circ\mu\cap\chi_M\circ\lambda(a), \chi_M\circ\mu\cap\lambda(b\beta c)\}\\
&= \sup_{x=a\alpha b\beta c} \min\{\chi_M\circ\mu\cap\lambda(a), \chi_M\circ\mu\cap\chi_M\circ\lambda(b\beta c)\}\\
&= \sup_{x=a\alpha b\beta c} \min\{\chi_M\circ\mu(a), \chi_M\circ\lambda(a)\}\}, \min\{\chi_M\circ\mu(b\beta c), \chi_M\circ\lambda(b\beta c)\}\}\\
&= \sup_{x=a\alpha\beta bc} \min\{\min\{\chi_M\circ\mu(a), \chi_M\circ\mu(b\beta c)\}, \min\{\chi_M\circ\lambda(a), \chi_M\circ\lambda(b\beta c)\}\}\\
&= \min\{\sup_{x=a\alpha b\beta c} \min\{\chi_M\circ\mu(a), \chi_M\circ\mu(b\beta c)\}, \sup_{x=a\alpha b\beta c} \min\{\chi_M\circ\lambda(a), \chi_M\circ\lambda(b\beta c)\}\}\\
&= \min\{\chi_M\circ\mu\circ\chi_M\circ\mu(x), \chi_M\circ\lambda\circ\chi_M\circ\lambda(x)\}\\
&= \chi_M\circ\mu\circ\chi_M\circ\mu\cap\chi_M\circ\lambda\circ\chi_M\circ\lambda(x).
\end{aligned}$$

So $\chi_M\circ\mu\cap\lambda\circ\chi_M\circ\mu\cap\lambda = \chi_M\circ\mu\circ\chi_M\circ\mu\cap\chi_M\circ\lambda\circ\chi_M\circ\lambda$. Hence

$$\chi_M\circ\mu\cap\lambda\circ\chi_M\circ\mu\cap\lambda = \chi_M\circ\mu\circ\chi_M\circ\mu\cap\chi_M\circ\lambda\circ\chi_M\circ\lambda \subseteq \mu\cap\lambda.$$

Therefore $\mu\cap\lambda$ is a left fuzzy quasi-interior ideal of M. This completes the proof. □

Theorem 4.21. *Let M be a $\Gamma-$semiring, let E be a parameter set and let $A \subseteq E$, $B \subseteq E$. If (f, A) and (g, B) are fuzzy soft left quasi-interior ideals of M, then $(f, A) \cap (g, B)$ is a fuzzy soft left quasi-interior ideal of$M$.*

Proof. Let (f, A) and (g, B) are fuzzy soft left quasi-interior ideals of$M$. By Definition 4.9, we have that $(f, A) \cap (g, B) = (h, C)$, where $C = A \cup B$.

Case (i): If $c \in A \setminus B$, then $h_c = f_c$. Thus h_c is a left quasi-interior ideal of S, since (f, A) is a fuzzy soft left quasi-interior ideal over M.

Case (ii): If $c \in B \setminus A$, then $h_c = g_c$. Thus h_c is a fuzzy left quasi-interior ideal of M, since $(g, B)$is a fuzzy soft left quasi-interior ideal over M.

Case (iii): If $c \in A \cap B$ and $x, y \in M, \alpha \in \Gamma$, then $h_c = f_c \cap g_c$. Thus by Theorem 4.20, h_c is a fuzzy left quasi-interior ideal of M. So $(f, A) \cap (g, B)$ is a fuzzy soft left quasi-interior ideal over M. □

Corollary 4.22. *If (f, A) and (g, B) are fuzzy soft right quasi-interior ideals of M, then $(f, A) \cap (g, B)$ is a fuzzy soft right quasi-interior ideal of M.*

Corollary 4.23. *Let (f, A) and (g, B) be fuzzy soft right ideal and fuzzy soft left ideal of M respectively. Then $(f, A) \cap (g, B)$ is a fuzzy soft quasi-interior ideal of M.*

Corollary 4.24. *Let (f, A) and (g, B) be fuzzy right ideal and fuzzy left ideal of M respectively. Then $(f, A) \cap (g, B)$ is a right fuzzy quasi-interior ideal of M.*

Proof of the following theorems are similar to theorems in [27]. Then we omit the proofs.

Theorem 4.25. *If (f, A) is a fuzzy soft quasi-ideal of regular Γ-semiring M, then (f, A) is a fuzzy soft ideal of M.*

Theorem 4.26. *A Γ-semiring M is a regular if and only if $f_a \circ g_b = (f, A) \cap (g, B)$ for any fuzzy soft right ideal (f, A) and fuzzy soft left ideal (g, B) of M.*

Theorem 4.27. *Let M be a regular Γ-semiring. Then (μ, A) is a fuzzy soft left quasi-interior ideal of M if and only if μ is a fuzzy soft quasi ideal of M.*

Proof. Let (μ, A) be a fuzzy soft left quasi-interior ideal of M and let $x \in M$. Then for each $a \in A$, $\chi_M \circ \mu_a \circ \chi_M \circ \mu_a \subseteq \mu_a$.

Suppose $\chi_M \circ \mu_a(x) > \mu_a(x)$ and $\mu_a \circ \chi_M(x) > \mu_a(x)$. Since M is a regular, there exist $y \in M, \alpha, \beta \in \Gamma$ such that $x = x\alpha y\beta x$, we have

$$\begin{aligned}\mu_a \circ \chi_M(x) &= \sup_{x=x\alpha y\beta x} \min\{\mu_a(x), \chi_M(y\beta x)\} \\ &= \sup_{x=x\alpha y\beta x} \min\{\mu_a(x), 1\} \\ &= \sup_{x=x\alpha y\beta x} \mu_a(x) \\ &> \mu_a(x).\end{aligned}$$

On the other hand, we get

$$\begin{aligned}\mu_a \circ \chi_M \circ \mu_a \circ \chi_M(x) &= \sup_{x=x\alpha y\beta x} \min\{\mu_a \circ \chi_M(x), \mu_a \circ \chi_M(y\beta x)\}) \\ &> \sup_{x=x\alpha y\beta x} \min\{\mu_a(x), \mu_a(y\beta x)\} \\ &= \mu_a(x),\end{aligned}$$

which is a contradiction. Then μ_a is a fuzzy quasi ideal of M. Thus (μ, A) is a fuzzy soft quasi-interior ideal of M. □

Corollary 4.28. *Let M be a regular Γ-semiring. Then (μ, A) is a fuzzy soft right quasi-interior ideal of M if and only if (μ, A) is a fuzzy soft quasi-interior ideal of M.*

Theorem 4.29. *Let M be a Γ-semiring. Then M is regular if and only if $A\Gamma B = A \cap B$ for any right ideal A and left ideal B of M.*

Theorem 4.30. *Let M be a Γ-semiring. Then M is regular if and only if $M\Gamma B\Gamma M\Gamma B = B(B\Gamma M\Gamma B\Gamma M = B)$ is a left(right) quasi-interior ideal of M.*

Proof. Let M be a regular Γ-semiring. Suppose B is a left quasi-interior ideal of M and let $x \in B$. Then $M\Gamma B\Gamma M\Gamma B \subseteq B$ and there exist $y \in M,\ \alpha, \beta \in$ such that

$$x = x\alpha y\beta x\alpha y\beta x \in M\Gamma B\Gamma M\Gamma B.$$

Thus $x \in M\Gamma B\Gamma M\Gamma B$. So $M\Gamma B\Gamma M\Gamma B = B$.
Similarly for right quasi interior ideal of M, we can prove $B\Gamma M\Gamma B\Gamma M = B$.

Conversely, suppose that $M\Gamma B\Gamma M\Gamma B = B$ for all left quasi-interior ideals B of M. Let $B = R\cap L$ and $C = R\Gamma L$, where R is a right ideal and L is a left ideal of M. Then B and C are quasi-interior ideals of M. Thus $(R\cap L)\Gamma M\Gamma(R\cap L)\Gamma M = R\cap L$. On the other hand, we get

$$\begin{aligned}
R \cap L &= (R \cap L)\Gamma M\Gamma(R \cap L)\Gamma M \\
&\subseteq R\Gamma M\Gamma L\Gamma M \\
&\subseteq R\Gamma L\Gamma M \\
R \cap L &= (R \cap L)\Gamma M\Gamma(R \cap L)\Gamma M \\
&\subseteq R\Gamma L\Gamma M\Gamma R\Gamma L\Gamma M \\
&\subseteq R\Gamma L \\
&\subseteq R \cap L \text{ [Since } R\Gamma L \subseteq L \text{ and } R\Gamma L \subseteq R].
\end{aligned}$$

So $R \cap L = R\Gamma L$. Hence M is a regular Γ-semiring. □

Theorem 4.31. *Let M be a $\Gamma-$semiring. Then M is a regular if and only if $\mu_a = \chi_M \circ \mu_a \circ \chi_M \circ \mu_a$ for any fuzzy soft left quasi-interior ideal (μ, A) of M.*

Proof. Suppose M is regular. Let (μ, A) be a fuzzy soft left quasi-interior ideal of M and let $x, y \in M, \alpha, \beta \in \Gamma$. Then $\chi_M \circ \mu_a \circ \chi_M \circ \mu_a \subseteq \mu_a$. Thus we get

$$\begin{aligned}
\chi_M \circ \mu_a \circ \chi_M \circ \mu_a(x) &= \sup_{x=x\alpha y\beta x} \{\min\{\chi_M \circ \mu_a(x), \chi_M \circ \mu_a(y\beta x)\}\} \\
&\geq \sup_{x=x\alpha y\beta x} \{\min\{\mu_a(x), \mu_a(x)\}\} \\
&= \mu_a(x).
\end{aligned}$$

So $\mu_a \subseteq \chi_M \circ \mu_a \circ \chi_M$. Hence $\chi_M \circ \mu_a \circ \chi_M \circ \mu_a = \mu_a$.

Conversely, suppose $\mu_a = \chi_M \circ \mu_a \circ \chi_M \circ \mu_a$ for any fuzzy soft quasi-interior ideal (μ, A) of the Γ-semiring M. Let B be a quasi-interior ideal of the Γ-semiring M. Then by Theorem 4.19, χ_B is a fuzzy quasi-interior ideal of M. Thus

$$\begin{aligned}
\text{Therefore } \chi_B &= \chi_M \circ \chi_B \circ \chi_M \circ \chi_B \\
&= \chi_{M\Gamma B\Gamma M\Gamma B}, \\
B &= M\Gamma B\Gamma M\Gamma B.
\end{aligned}$$

So by Theorem 4.30, M is a regular $\Gamma-$ semiring . □

Theorem 4.32. *Let M be a Γ-semiring. Then M is a regular if and only if $\mu_a \cap \gamma_b \subseteq \mu_a \circ \gamma_b \circ \mu_a \circ \gamma_b$, for all $a \in A, b \in B$, fuzzy soft left quasi-interior ideal (μ, A) and every fuzzy soft ideal (γ, B) of M.*

Proof. Suppose M be regular and let $x \in M$. Then there exist $y \in M, \alpha, \beta \in \Gamma$ such that $x = x\alpha y\beta x$. Thus we have

$$\begin{aligned}\mu_a \circ \gamma_b \circ \mu_a \circ \gamma_b(x) &= \sup_{x=x\alpha y\beta x} \{\min\{\mu_a \circ \gamma_b(x\alpha y), \mu_a \circ \gamma_b(x)\}\} \\ &= \min\{\sup_{x\alpha y = x\alpha y\beta x\alpha y} \{\min\{\mu_a(x), \gamma_b(y\beta x\alpha y)\}, \\ &\qquad \sup_{x\alpha y = x\alpha y\beta x\alpha y} \{\min\{\mu_a(x), \gamma_b(y\beta x\alpha y)\}\}\} \\ &\geq \min\{\min\{\mu_a(x), \gamma_b(x)\}, min\{\mu_a(x), \gamma_b(x)\}\} \\ &= \min\{\mu_a(x), \gamma_b(x)\} = \mu_a \cap \gamma_b(x).\end{aligned}$$

So $(\mu_a) \cap (\gamma_B) \subseteq \mu_a \circ \gamma_b \circ \mu_a \circ \gamma_b$.

Conversely, suppose that the necessary condition holds. Let (μ, A) be a fuzzy soft left quasi-interior ideal of M. Then we get

$$\mu_a \cap \chi_M \subseteq \chi_M \circ \mu_a \circ \chi_M \circ \mu_a, \mu_a \subseteq \chi_M \circ \mu_a \circ \chi_M \circ \mu_a.$$

Thus by Theorem 4.31, M is regular. □

Corollary 4.33. *Let M be a Γ-semiring . Then M is regular if and only if $\mu_a \cap \gamma_b \subseteq \mu_a \circ \gamma_b \circ \mu_a \circ \gamma_b$ for all $a \in A, b \in B$, fuzzy soft right quasi-interior ideal (μ, A) and every fuzzy soft ideal (γ, B) of M.*

5. Conclusion

In this paper, we introduced the notion of fuzzy soft right (left) quasi-interior ideal, fuzzy soft quasi-interior ideals and studied the properties of fuzzy soft quasi-interior ideals ideals of Γ-semiring . Further we discussed relation between these ideals. Characterized the regular Γ-semiring in terms of fuzzy soft right(left)quasi-interior ideals of a Γ-semiring and studied some of their algebraical properties.

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