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Abstract. In this paper, a new form of somewhat fuzzy continuity, called A-somewhat fuzzy continuity between fuzzy topological spaces, is introduced. Several characterizations of these functions are obtained and a condition for an fuzzy α-continuous function to become an A-somewhat fuzzy continuous function is established. It is shown that A-somewhat fuzzy continuous function from the fuzzy hyperconnected space into another fuzzy topological space is not an somewhat fuzzy continuous function. The conditions by means of A-somewhat fuzzy continuous functions for the fuzzy hyperconnected and fuzzy strongly irresolvable spaces to become fuzzy Baire spaces, are also obtained.

2020 AMS Classification: 54A40, 03E72

Keywords: Fuzzy dense set, Fuzzy somewhere dense set, Fuzzy cs dense set, Fuzzy β-open set, Fuzzy nodec space, Fuzzy α-continuous function, Fuzzy hyperconnected space, Fuzzy Baire space.

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1. Introduction

In 1965, Zadeh [1] introduced the concept of fuzzy sets as a new approach for modelling uncertainties. The potential of fuzzy notion was realized by the mathematicians and has successfully been applied in all branches of Mathematics. Topology provided the most natural framework for the concepts of fuzzy sets to flourish. The concept of fuzzy topological space was introduced by Chang [2] in 1968. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. In recent years, the topological space theory has been embedding in the soft set theory to obtain some interesting applications [3, 4]. Lee et al. [5] defined an Octahedron Set composed of an interval-valued fuzzy set, an intuitionistic set and a fuzzy set that will provide nice information about uncertainty and vagueness. Recently, Şenel et al. [6] applied the concept of Octahedron Sets to multi-criteria group decision-making problems.

In the recent years, a considerable amount of research has been done on many types of continuity in general topology. The class of somewhat continuous mappings was first introduced by Gentry and Hoyle [7]. Later, the concept of "somewhat" in classical topology has been extended to fuzzy topological spaces. The notions of somewhat fuzzy continuous mappings and somewhat fuzzy open mappings on fuzzy topological spaces were introduced and studied by Thangaraj and Balasubramanian in [8]. Baker [9] introduced an alternate form of somewhat continuity, called *A-somewhat continuity*, in general topology. Motivated on these lines, a new form of somewhat fuzzy continuity, called A-somewhat fuzzy continuity between fuzzy topological spaces is introduced and relationships between A-somewhat continuity and other generalized continuity are investigated in this paper. It is shown that A-somewhat fuzzy continuity is independent of somewhat fuzzy continuity. Several characterizations of these functions are developed. A condition for an fuzzy α-continuous function to become an A-somewhat fuzzy continuous function is established. It is shown that A-somewhat fuzzy continuous function from the fuzzy hyperconnected space into another fuzzy topological space is not an somewhat fuzzy continuous function. The conditions for the fuzzy hyperconnected and fuzzy strongly irresolvable spaces to become fuzzy Baire spaces, are also obtained, in this paper.

2. Preliminaries

In order to make the exposition self-contained, some basic notions and results used in the sequel, are given. In this work by (X, T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). Let X be a non-empty set and I the unit interval $[0, 1]$. A fuzzy set λ in X is a mapping from X into I. The fuzzy set 0_X is defined as $0_X(x) = 0$, for all $x \in X$ and the fuzzy set 1_X is defined as $1_X(x) = 1$, for all $x \in X$.

Definition 2.1 ([2])**.** Let λ be any fuzzy set in the fuzzy topological space (X, T). Then the *fuzzy interior*, the *fuzzy closure* and the *fuzzy complement* of λ are defined respectively as follows:

(i) $int(\lambda) = \vee\{\mu \mid \mu \leq \lambda,\ \mu \in T\}$,
(ii) $cl(\lambda) = \wedge\{\mu \mid \lambda \leq \mu,\ 1 - \mu \in T\}$,
(iii) $\lambda'(x) = 1 - \lambda(x)$ for all $x \in X$.

The *union* $\psi = \vee_i(\lambda_i)$ and the *intersection* $\delta = \wedge_i(\lambda_i)$, are defined respectively, for the family $\{\lambda_i \mid i \in I\}$ of fuzzy sets in (X, T) as follows:

(iv) $\psi(x) = \sup_i\{\lambda_i(x) \mid x \in X\}$,
(v) $\delta(x) = \inf_i\{\lambda_i(x) \mid x \in X\}$.

Lemma 2.2 ([10])**.** *For a fuzzy set λ of a fuzzy topological space X, we have*
(1) $1 - int(\lambda) = cl(1 - \lambda)$,
(2) $1 - cl(\lambda) = int(1 - \lambda)$.

Definition 2.3. A fuzzy set λ in a fuzzy topological space (X, T) is said to be:

(i) *fuzzy regular-open*, if $\lambda = intcl(\lambda)$ and *fuzzy regular-closed* [10], if $\lambda = clint(\lambda)$,

(ii) *fuzzy pre-open*, if $\lambda \leq intcl(\lambda)$ and *fuzzy pre-closed* [11], if $clint(\lambda) \leq \lambda$,
(iii) *fuzzy semi-open*, if $\lambda \leq clint(\lambda)$ and *fuzzy semi-closed* [10], if $intcl(\lambda) \leq \lambda$,
(iv) *fuzzy α-open*, if $\lambda \leq intclint(\lambda)$ and *α-closed* [12], if $clintcl(\lambda) \leq \lambda$,
(v) *fuzzy β-open*, if $\lambda \leq clintcl(\lambda)$ and fuzzy β-closed [13], if $intclint(\lambda \leq \lambda$,
(vi) *fuzzy G_δ-set* [10], if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where $\lambda_i \in T$ for $i \in I$,
(vii) *fuzzy F_σ-set* [10], if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $1 - \lambda_i \in T$ for $i \in I$.

Definition 2.4 ([8])**.** A fuzzy set λ in a fuzzy topological space (X, T) is called *fuzzy dense*, if there exists no fuzzy closed set μ in (X, T) such that $\lambda < \mu < 1$. That is, $cl(\lambda) = 1$ in (X, T).

Definition 2.5 ([8])**.** A fuzzy set λ in a fuzzy topological space (X, T) is called *fuzzy nowhere dense*, if there exists no non-zero fuzzy open set μ in (X, T) such that $\mu < cl(\lambda)$. That is, $intcl(\lambda) = 0$ in (X, T).

Definition 2.6 ([8])**.** A fuzzy set λ in a fuzzy topological space (X, T) is called a *fuzzy first category set*, if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X, T).

Any other fuzzy set in (X, T) is said to be of *fuzzy second category*.

Definition 2.7 ([14])**.** Let λ be a fuzzy first category set in the fuzzy topological space (X, T). Then $1 - \lambda$ is called an *fuzzy residual set* in (X, T).

Definition 2.8. Let (X, T) be the fuzzy topological space and (X, T) is said called a:

(i) *Baire space* [14], if $int\,(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where (λ_i)'s are fuzzy nowhere dense sets in (X, T),
(ii) *fuzzy hyperconnected space* [15], if every non-null fuzzy open subset of (X, T) is fuzzy dense in (X, T),
(iii) *fuzzy submaximal space* [16], if for each fuzzy set λ in (X, T) such that $cl(\lambda) = 1$, $\lambda \in T$,
(iv) *almost resolvable space* [17], if $\vee_{i=1}^{\infty}(\lambda_i) = 1_X$ where the fuzzy sets (λ_i)'s in (X, T) are such that $int(\lambda_i) = 0$, otherwise (X, T) is called an *fuzzy almost irresolvable space*,
(v) *fuzzy first category space* [14], if $1_X = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X, T), otherwise (X, T) is said to be of *fuzzy second category*,
(vi) *fuzzy open hereditarily irresolvable space* [18], provided that if $intcl(\lambda) \neq 0$ for any non-zero fuzzy set λ defined on X, then $int(\lambda) \neq 0$ in (X, T),
(vii) *fuzzy strongly irresolvable space* [17], if for every fuzzy dense set λ in (X, T), $clint(\lambda) = 1$ in (X, T),
(viii) *fuzzy nodec space* [19], if each non-zero fuzzy nowhere dense set is fuzzy closed in (X, T),
(ix) *fuzzy perfectly disconnected space* [20], if for any two non-zero fuzzy sets λ and μ defined on X with $\lambda \leq 1 - \mu$, $cl(\lambda) \leq 1 - cl(\mu)$ in (X, T).

Definition 2.9 ([18])**.** Let (X, T) be the fuzzy topological space. Then a fuzzy set λ defined on X is called an *fuzzy somewhere dense set*, if $intcl(\lambda) \neq 0$ in (X, T). That is, λ is an fuzzy somewhere dense set in (X, T), if there exists an non-zero fuzzy open set μ in (X, T) such that $\mu \leq cl(\lambda)$.

If λ is an fuzzy somewhere dense set in the fuzzy topological space (X,T), then $1-\lambda$ is called an *fuzzy complement* of fuzzy somewhere dense set in (X,T). It is to be denoted as fuzzy cs dense set in (X,T) [21].

Definition 2.10 ([8]). Let (X,T) and (Y,S) be any two fuzzy topological spaces. Then A function $f:(X,T)\to(Y,S)$ is called an *somewhat fuzzy continuous function*, if $\lambda\in S$ and $f^{-1}(\lambda)\neq 0$, there exists an non-zero fuzzy open set μ in (X,T) such that $\mu\leq f^{-1}(\lambda)$. That is, $int[f^{-1}(\lambda)]\neq 0$ in (X,T).

Definition 2.11 ([22]). Let (X,T) be a fuzzy topological space. Then a fuzzy set λ is called an *fuzzy resolvable set*, if for each fuzzy closed set μ in (X,T), $\{cl(\mu\wedge\lambda)\wedge cl[\mu\wedge(1-\lambda)]\}$ is an fuzzy nowhere dense set in (X,T). In other words, λ is a fuzzy resolvable set in (X,T) ,if $intcl\{cl(\mu\wedge\lambda)\wedge cl[\mu\wedge(1-\lambda)]\}=0$, where $1-\mu\in T$.

Definition 2.12 ([23]). Let (X,T) and (Y,S) be any two fuzzy topological spaces. Then a function $f:(X,T)\to(Y,S)$ is called a *fuzzy contra β-continuous function*, if for each fuzzy open set λ in (Y,S), $f^{-1}(\lambda)$ is an fuzzy β-closed set in (X,T).

Theorem 2.13 ([24]). *If λ is an fuzzy somewhere dense set in the fuzzy hyperconnected space (X,T), then λ is a fuzzy dense set in (X,T).*

Theorem 2.14 ([24]). *If λ is an fuzzy somewhere dense set in the fuzzy hyperconnected space (X,T), then λ is an fuzzy β-open set in (X,T).*

Theorem 2.15 ([21]). *If λ is an non-zero fuzzy β-open set in the fuzzy topological space (X,T), then λ is an fuzzy somewhere dense set in (X,T).*

Theorem 2.16 ([24]). *If the fuzzy set λ is an fuzzy cs dense set in the fuzzy hyperconnected space (X,T), then $int(\lambda)=0$, in (X,T).*

Theorem 2.17 ([24]). *If λ is an fuzzy somewhere dense set in the fuzzy hyperconnected and fuzzy open hereditarily irresolvable space, then the fuzzy cs dense set $1-\lambda$ is an fuzzy nowhere dense set in (X,T).*

Theorem 2.18 ([21]). *A fuzzy set λ in the fuzzy topological space (X,T) is an fuzzy cs dense set if and only if there exists an fuzzy closed set μ in (X,T) such that $int(\lambda)\leq\mu$.*

Theorem 2.19 ([24]). *λ is an fuzzy somewhere dense set in the fuzzy strongly irresolvable space (X,T) if and only if $int(\lambda)\neq 0$ in (X,T).*

Theorem 2.20 ([24]). *If $cl\left(\wedge_{i=1}^{\infty}(\lambda_i)\right)=1$, where (λ_i)'s are fuzzy somewhere dense sets in the fuzzy hyperconnected and fuzzy open hereditarily irresolvable space (X,T), then (X,T) is an fuzzy Baire space.*

Theorem 2.21 ([25]). *If the fuzzy topological space (X,T) is an fuzzy open hereditarily irresolvable space, then $int(\lambda)=0$ for any non-zero fuzzy set λ in (X,T) implies that $intcl(\lambda)=0$.*

Theorem 2.22 ([10]). *In a fuzzy space X,*

(1) *the closure of a fuzzy open set is a fuzzy regular closed set,*
(2) *the interior of a fuzzy closed set is a fuzzy regular open set.*

Theorem 2.23 ([26]). *If $\lambda \leq \mu$, where μ is a fuzzy set and λ is a fuzzy dense set in a fuzzy strongly irresolvable space (X, T), then $1 - \mu$ is a fuzzy nowhere dense set in (X, T).*

Theorem 2.24 ([24]). *If (λ_i)'s ($i = 1$ to ∞) are fuzzy cs dense sets in the fuzzy topological space (X, T), then there exists an fuzzy open set λ and fuzzy F_σ-set μ in (X, T) such that $\lambda \leq \mu$, where $\lambda = \vee_{i=1}^{\infty} (int(\lambda_i))$ and $\mu = \vee_{i=1}^{\infty}(\mu_i)$, $1 - \mu_i \in T$.*

Theorem 2.25 ([14]). *If a fuzzy topological space (X, T) is the fuzzy Baire space, then (X, T) is the fuzzy second category space.*

Theorem 2.26 ([24]). *If λ is an fuzzy closed set in the fuzzy topological space (X, T), then λ is an fuzzy cs dense set in (X, T).*

Theorem 2.27 ([24]). *If $\vee_{i=1}^{\infty}(\lambda_i) \neq 1$, where (λ_i)'s are fuzzy cs dense sets in the fuzzy hyperconnected space (X, T), then (X, T) is the fuzzy almost irresolvable space.*

Theorem 2.28 ([21]). *If λ is a fuzzy somewhere dense set in a fuzzy topological space (X, T), then there exists an fuzzy regular closed set η in (X, T) such that $\eta \leq cl(\lambda)$.*

Theorem 2.29 ([20]). *If λ is a fuzzy set in a fuzzy perfectly disconnected space (X, T), then $int(\lambda)$ is a fuzzy closed set in (X, T).*

Theorem 2.30 ([27]). *If (λ_i)'s ($i = 1$ to ∞) are fuzzy somewhere dense sets in a fuzzy perfectly disconnected space (X, T), then there exists an fuzzy G_δ-set η in (X, T) such that $\eta \leq \wedge_{i=1}^{\infty} (cl(\lambda_i))$.*

Theorem 2.31 ([20]). *If λ is an fuzzy open set in the fuzzy perfectly disconnected space (X, T), then $intcl(\lambda)$ is an fuzzy closed set in (X, T).*

Theorem 2.32 ([28]). *If λ is a non-zero fuzzy β-open set in a fuzzy hyperconnected space (X, T), then there exists a fuzzy resolvable set μ in (X, T) such that $\mu \leq cl[\lambda]$.*

Theorem 2.33 ([22]). *If $int\,(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where (λ_i)'s are fuzzy resolvable sets in a fuzzy topological space (X, T), then (X, T) is a fuzzy Baire space.*

3. A-somewhat fuzzy continuous functions

Definition 3.1. Let (X, T) and (Y, S) be any two fuzzy topological spaces. Then a function $f : (X, T) \to (Y, S)$ is called an *A-somewhat fuzzy continuous function*, if for each fuzzy closed set μ in (Y, S) such that $f^{-1}(\mu) \neq 0$, there exists an non-zero fuzzy closed set δ in (X, T) such that $\delta \leq f^{-1}(\mu)$.

Clearly, the fuzzy continuous function between fuzzy topological spaces is the A-somewhat fuzzy continuous function. For, if μ is an fuzzy closed set in (Y, S) and $f : (X, T) \to (Y, S)$ is an fuzzy continuous function from (X, T) into (Y, S), then $f^{-1}(\mu)$ is an fuzzy closed set in (X, T). Thus $cl\left(f^{-1}(\mu)\right) = f^{-1}(\mu)$ implies that $cl\left(f^{-1}(\mu)\right) \leq f^{-1}(\mu)$. So $f : (X, T) \to (Y, S)$ is an A-somewhat fuzzy continuous function.

But the converse need not be true. That is, the A-somewhat fuzzy continuous function need not be the fuzzy continuous function.

Example 3.2. Let $X = \{a, b, c\}$. The fuzzy sets λ, μ and δ are defined on X as follows:

$\lambda : X \to [0, 1]$ is defined as $\lambda(a) = 0.3$; $\lambda(b) = 0.2$; $\lambda(c) = 0$,
$\mu : X \to [0, 1]$ is defined as $\mu(a) = 0.7$; $\mu(b) = 0.6$; $\mu(c) = 1$,
$\delta : X \to [0, 1]$ is defined as $\delta(a) = 1$; $\delta(b) = 0.6$; $\delta(c) = 0.2$.

Then clearly, $T = \{0, \lambda, \mu, 1\}$ and $S = \{0, \delta, 1\}$ are fuzzy topologies on X. Define the function $g : (X, T) \to (X, S)$ by $g(a) = c$, $g(b) = b$, $g(c) = a$. On computation, one can find that for the fuzzy closed set $1 - \delta$ in (X, S), there exists an fuzzy closed set $1 - \mu$ in (X, T) such that $1 - \mu \leq g^{-1}(1 - \delta)$. Thus the function $g : (X, T) \to (X, S)$ is the A-somewhat fuzzy continuous function from (X, T) into (X, S). Also on computation, one can find that for the fuzzy open set δ in (X, S), there exists no fuzzy open set η in (X, T) such that $\eta = g^{-1}(\delta)$ and hence the function $g : (X, T) \to (X, S)$ is not the fuzzy continuous function from (X, T) into (X, S).

Remark 3.3. Although the definition of A-somewhat fuzzy continuity is analogous to that of somewhat fuzzy continuity, the two concepts are independent.

The following example shows that the somewhat fuzzy continuous function need not be the A-somewhat fuzzy continuous function.

Example 3.4. Let μ_1, μ_2 and μ_3 be fuzzy sets of I, where $I = [0, 1]$ defined as

$$\mu_1(x) = \begin{cases} 0, & 0 \leq x \leq \frac{1}{2} \\ 2x - 1, & \frac{1}{2} \leq x \leq 1, \end{cases}$$

$$\mu_2(x) = \begin{cases} 1, & 0 \leq x \leq \frac{1}{4} \\ -4x + 2, & \frac{1}{4} \leq x \leq \frac{1}{2} \\ 0, & \frac{1}{2} \leq x \leq \frac{1}{4}, \end{cases}$$

$$\mu_3(x) = \begin{cases} 1, & 0 \leq x \leq \frac{1}{4} \\ \frac{2}{3}(2 - 2x), & \frac{1}{4} \leq x \leq 1. \end{cases}$$

Let $T = \{0, \mu_1, \mu_2, \mu_1 \vee \mu_2, 1\}$ and $S = \{0, \mu_3, 1\}$. Then T and S are fuzzy topologies on I.

Consider the function $f : (I, T) \to (I, S)$ defined by $f(x) = \frac{x}{2}$, for each $x \in I$. On computation, one can find that for the fuzzy closed set $1 - \mu_3$ in (I, S), there is no fuzzy closed set δ $(= 1 - \mu_1, 1 - \mu_2, 1 - [\mu_1 \vee \mu_2])$ in (I, T) such that $\delta \leq f^{-1}(1 - \mu_3)$. Hence the function $f : (I, T) \to (I, S)$ is not the A-somewhat fuzzy continuous function from (I, T) into (I, S). Also on computation, one can find that for the fuzzy open set μ_3 in (I, S), there exists an fuzzy open set $\mu_1 \vee \mu_2 \leq f^{-1}(\mu_3)$ and hence the function $f : (I, T) \to (I, S)$ is the somewhat fuzzy continuous function from (I, T) into (I, S).

The following example shows that the A-somewhat fuzzy continuous function need not be the somewhat fuzzy continuous function.

Example 3.5. Let $X = \{a, b, c\}$. The fuzzy sets λ, μ and δ are defined on X as follows:

$\lambda : X \to [0, 1]$ is defined as $\lambda(a) = 0.3$; $\lambda(b) = 0.2$; $\lambda(c) = 0$,

$\mu : X \to [0,1]$ is defined as $\mu(a) = 0.7$; $\mu(b) = 0.6$; $\mu(c) = 1$,
$\delta : X \to [0,1]$ is defined as $\delta(a) = 1$; $\delta(b) = 0.6$; $\delta(c) = 0.2$.
Then clearly, $T = \{0, \lambda, \mu, 1\}$ and $S = \{0, \delta, 1\}$ are fuzzy topologies on X. Define the function $g : (X,T) \to (X,S)$ by $g(a) = c$, $g(b) = b$, $g(c) = a$. On computation one can find that for the fuzzy closed set $1 - \delta$ in (X,S), there exists an fuzzy closed set $1 - \mu$ in (X,T) such that $1 - \mu \leq g^{-1}(1 - \delta)$. Thus the function $g : (X,T) \to (X,S)$ is the A-somewhat fuzzy continuous function from (X,T) into (X,S). Also on computation one can find that for the fuzzy open set δ in (X,S), there exists no fuzzy open set η in (X,T) such that $\eta \leq g^{-1}(\delta)$ and hence the function $g : (X,T) \to (X,S)$ is not the somewhat fuzzy continuous function from (X,T) into the fuzzy (X,S).

The following example shows that the function defined between fuzzy topological spaces is neither A-somewhat fuzzy continuous function nor the somewhat fuzzy continuous function.

Example 3.6. Let $X = \{a, b, c\}$. The fuzzy sets λ, μ and γ are defined on X as follows:
$\lambda : X \to [0,1]$ is defined as $\lambda(a) = 0.3$; $\lambda(b) = 0.2$; $\lambda(c) = 0$,
$\mu : X \to [0,1]$ is defined as $\mu(a) = 0.7$; $\mu(b) = 0.6$; $\mu(c) = 1$,
$\gamma : X \to [0,1]$ is defined as $\gamma(a) = 0$; $\gamma(b) = 0.4$; $\gamma(c) = 0.8$.
Then clearly, $T = \{0, \gamma, 1\}$ and $S = \{0, \lambda, \mu, 1\}$ are fuzzy topologies on X. Define the function $h : (X,T) \to (X,S)$ by $h(a) = c$, $h(b) = b$, $h(c) = a$. On computation, one can find that for the fuzzy closed set $1 - \mu$ in (X,S), there is an fuzzy closed set $1 - \gamma$ in (X,T) such that $1 - \gamma \geq h^{-1}(1 - \mu)$. Hence $h : (X,T) \to (X,S)$ is not the A-somewhat fuzzy continuous function from (X,T) into (X,S). Also on computation, one can find that for the fuzzy open set λ in (X,S), there exists an fuzzy open set γ in (X,T) such that $\gamma \geq h^{-1}(\lambda)$. Thus the function $h : (X,T) \to (X,S)$ is not the somewhat fuzzy continuous function from (X,T) into (X,S).

Proposition 3.7. *If the function $f : (X,T) \to (Y,S)$ is A-somewhat fuzzy continuous function from the fuzzy topological space (X,T) into the fuzzy topological space (Y,S) and μ is an non-zero fuzzy closed set in (Y,S), then there exists an fuzzy regular-open set η in (X,T) such that $\eta \leq intcl[f^{-1}(\mu)]$.*

Proof. Let μ be an non-zero fuzzy closed set in (Y,S) such that $f^{-1}(\mu) \neq 0$. Since $f : (X,T) \to (Y,S)$ is the A-somewhat fuzzy continuous function from (X,T) into (Y,S), there exists an non-zero fuzzy closed set δ in (X,T) such that $\delta \leq f^{-1}(\mu)$. Then $cl(\delta) \leq cl[f^{-1}(\mu)]$ in (X,T). Since δ is an fuzzy closed set, $cl(\delta) = \delta$. Thus $\delta \leq cl[f^{-1}(\mu)]$ in (X,T). So $int(\delta) \leq intcl[f^{-1}(\mu)]$. Let $\eta = int(\delta)$. By theorem 2.22, the interior of an fuzzy closed set is an fuzzy regular open set in fuzzy topological spaces, $int(\delta)$ is an fuzzy regular open set in (X,T). Hence for the non-zero fuzzy closed set μ in (Y,S), there exists an fuzzy regular-open set η in (X,T) such that $\eta \leq intcl[f^{-1}(\mu)]$. □

Proposition 3.8. *If the function $f : (X,T) \to (Y,S)$ is A-somewhat fuzzy continuous function from the fuzzy topological space (X,T) into the fuzzy topological space (Y,S) and μ is an non-zero fuzzy closed set in (Y,S), then $f^{-1}(\mu)$ is an fuzzy somewhere dense set in (X,T).*

Proof. Let μ be an non-zero fuzzy closed set in (Y, S) such that $f^{-1}(\mu) \neq 0$. Since $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from (X, T) into (Y, S), by Proposition 3.7, there exists an fuzzy regular-open set η in (X, T) such that $\eta \leq intcl[f^{-1}(\mu)]$. Since an fuzzy regular-open set is an fuzzy open set in fuzzy topological spaces, η is an fuzzy open set in (X, T) and then $intcl[f^{-1}(\mu)] \neq 0$, in (X, T). Then $f^{-1}(\mu)$ is an fuzzy somewhere dense set in (X, T). □

Proposition 3.9. *For a function $f : (X, T) \to (Y, S)$, the following are equivalent:*

(1) *$f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function,*
(2) *for each fuzzy open set λ in (Y, S) such that $f^{-1}(\lambda) \neq 1$, there exists an fuzzy open set $\eta \neq 1$, in (X, T) such that $f^{-1}(\lambda) \leq \eta$.*

Proof. (1) $\Longrightarrow$ (2): Let λ be an non-zero fuzzy open set in (Y, S) such that $f^{-1}(\lambda) \neq 1$. Then $1 - \lambda$ is an fuzzy closed set in (Y, S) such that $f^{-1}(1 - \lambda) \neq 0$. Since $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from (X, T) into (Y, S), there exists an non-zero fuzzy closed set $\delta(\neq 0)$ in (X, T) such that $\delta \leq f^{-1}(1 - \lambda)$. Thus $\delta \leq 1 - f^{-1}(\lambda)$ and this implies that $f^{-1}(\lambda) \leq 1 - \delta$. Let $\eta = 1 - \delta$. Since $\delta \neq 0$, $\eta \neq 1$ in (X, T). Then for the fuzzy open set λ in (Y, S) such that $f^{-1}(\lambda) \neq 1$, there exists an fuzzy open set $\eta \neq 1$, in (X, T) such that $f^{-1}(\lambda) \leq \eta$.

(2) $\Longrightarrow$ (1): Let μ be an non-zero fuzzy closed set in (Y, S) such that $f^{-1}(\mu) \neq 0$. Then $1 - \mu$ is an fuzzy open set in (Y, S) such that $f^{-1}(1 - \mu) \neq 1$. Then by (2), there exists an fuzzy open set $\eta(\neq 1)$ in (X, T) such that $f^{-1}(1 - \mu) \leq \eta$. This implies that $1 - f^{-1}(\mu) \leq \eta$. Thus $1 - \eta \leq f^{-1}(\mu)$. Since η is an fuzzy open set in (X, T), $1 - \eta$ is an fuzzy closed set in (X, T). Let $\delta = 1 - \eta$ and $\eta \neq 1$, implies that $\delta \neq 0$ in (X, T). Then for the non-zero fuzzy closed set μ in (Y, S), there exists an fuzzy closed set $\delta \neq 0$ in (X, T) such that $\delta \leq f^{-1}(\mu)$. Thus $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function. □

Proposition 3.10. *If the function $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from the fuzzy topological space (X, T) into the fuzzy topological space (Y, S) and λ is an non-zero fuzzy open set in (Y, S) such that $f^{-1}(\lambda) \neq 1$, then $f^{-1}(\lambda)$ is an fuzzy cs dense set in (X, T).*

Proof. Let λ be an non-zero fuzzy open set in (Y, S) such that $f^{-1}(\lambda) \neq 1$. Then $1-\lambda$ is an fuzzy closed set in (Y, S) such that $f^{-1}(1-\lambda) \neq 0$. Since $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from (X, T) into (Y, S), by Proposition 3.8, $f^{-1}(1 - \lambda)$ is an fuzzy somewhere dense set in (X, T) . Since $f^{-1}(1 - \lambda) = 1 - f^{-1}(\lambda)$, $1 - f^{-1}(\lambda)$ is an fuzzy somewhere dense set in (X, T). Thus $f^{-1}(\lambda)$ is an fuzzy cs dense set in (X, T). □

The following proposition gives a condition for fuzzy α-continuous functions to become A-somewhat fuzzy continuous functions.

Proposition 3.11. *If $f : (X, T) \to (Y, S)$ is an fuzzy α-continuous function from the fuzzy topological space (X, T) in which there is no fuzzy open and fuzzy dense set into the fuzzy topological space (Y, S), then $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from (X, T) into (Y, S).*

Proof. Let λ be an non-zero fuzzy open set in (Y, S) such that $f^{-1}(\lambda) \neq 1$. Since the function $f : (X, T) \to (Y, S)$ is an fuzzy α-continuous function from (X, T) into (Y, S), $f^{-1}(\lambda)$ is an fuzzy α-open set in (X, T). This implies that $f^{-1}(\lambda) \leq intclint[f^{-1}(\lambda)]$. Since $f^{-1}(\lambda) \neq 1$, $int[f^{-1}(\lambda)] \neq 1$. Since there is no fuzzy open and fuzzy dense set in (X, T), $cl(int[f^{-1}(\lambda)]) \neq 1$ and hence $int\{cl(int[f^{-1}(\lambda)])\} \neq 1$. Let $\eta = intclint[f^{-1}(\lambda)]$. Then $\eta \neq 1$. Thus for the fuzzy open set λ in (Y, S) such that $f^{-1}(\lambda) \neq 1$, there exists an fuzzy open set $\eta (\neq 1)$, in (X, T) such that $f^{-1}(\lambda) \leq \eta$. So by Proposition 3.9, $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from (X, T) into (Y, S). □

Proposition 3.12. *If $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from the fuzzy hyperconnected space (X, T) into the fuzzy topological space (Y, S) and μ is an non-zero fuzzy closed set in (Y, S) such that $f^{-1}(\mu) \neq 1$, then*

(1) *$cl[f^{-1}(\mu)] = 1$ in (X, T),*
(2) *$int[f^{-1}(1 - \mu)] = 0$ in (X, T).*

Proof. (1) Let μ be an non-zero fuzzy closed set in (Y, S) such that $f^{-1}(\mu) \neq 0$. Since $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from (X, T) into (Y, S), by Proposition 3.8, $f^{-1}(\mu)$ is an fuzzy somewhere dense set in (X, T). Since (X, T) is the fuzzy hyperconnected space, by Theorem 2.13, the fuzzy somewhere dense set $f^{-1}(\mu)$ is an fuzzy dense set in (X, T). Then $cl[f^{-1}(\mu)] = 1$ in (X, T).

(2) From (1), $cl[f^{-1}(\mu)] = 1$. Then $1 - cl[f^{-1}(\mu)] = 0$, in (X, T). Thus by Lemma 2.2, $int[1 - f^{-1}(\mu)] = 0$. So $int[f^{-1}(1 - \mu)] = 0$ in (X, T). □

Proposition 3.13. *If $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from the fuzzy hyperconnected space (X, T) into the fuzzy topological space (Y, S) and μ is an non-zero fuzzy closed set in (Y, S) such that $f^{-1}(\mu) \neq 0$, then $f^{-1}(\mu)$ is an fuzzy β-open set in (X, T).*

Proof. Let μ be an non-zero fuzzy closed set in (Y, S) such that $f^{-1}(\mu) \neq 0$. Since $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from (X, T) into (Y, S), by Proposition 3.8, $f^{-1}(\mu)$ is an fuzzy somewhere dense set λ in (X, T). Since (X, T) is the fuzzy hyperconnected space, by Theorem 2.14, $f^{-1}(\mu)$ is an fuzzy β-open set in (X, T). □

Proposition 3.14. *If $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from the fuzzy hyperconnected space (X, T) into the fuzzy topological space (Y, S), then f is the fuzzy contra β-continuous function from (X, T) into (Y, S).*

Proof. Let λ be an non-zero fuzzy open set in (Y, S) such that $f^{-1}(\lambda) \neq 1$. Then $1 - \lambda$ is an non-zero fuzzy closed set in (Y, S) such that $f^{-1}(1 - \lambda) \neq 0$. Since $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from (X, T) into (Y, S), by Proposition 3.13, $f^{-1}(1 - \lambda)$ is an fuzzy β-open set in (X, T). Since $f^{-1}(1 - \lambda) = 1 - f^{-1}(\lambda)$, $f^{-1}(\lambda)$ is an fuzzy β-closed set in (X, T). Thus $f : (X, T) \to (Y, S)$ is the fuzzy contra β-continuous function from (X, T) into (Y, S). □

Proposition 3.15. *If $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from the fuzzy hyperconnected space (X, T) into the fuzzy topological space (Y, S) and λ is an non-zero fuzzy open set in (Y, S) such that $f^{-1}(\lambda) \neq 1$, then $int[f^{-1}(\lambda)] = 0$, in (X, T).*

Proof. Let λ be an non-zero fuzzy open set in (Y, S) such that $f^{-1}(\lambda) \neq 1$. Since $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from the fuzzy hyperconnected space (X, T) into (Y, S), by Proposition 3.10, $f^{-1}(\lambda)$ is an fuzzy cs dense set in (X, T). Thus by Theorem 2.16, for the fuzzy cs dense set $f^{-1}(\lambda)$, $int[f^{-1}(\lambda)] = 0$ in (X, T). □

The following proposition shows that the A-somewhat fuzzy continuous function from the fuzzy hyperconnected space into another fuzzy topological space is not the somewhat fuzzy continuous function and not an fuzzy continuous function.

Proposition 3.16. *If $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from the fuzzy hyper connected space (X, T) into the fuzzy topological space (Y, S), then*

(1) *$f : (X, T) \to (Y, S)$ is not an somewhat fuzzy continuous function,*
(2) *$f : (X, T) \to (Y, S)$ is not an fuzzy continuous function.*

Proof. (1) Let λ be an non-zero fuzzy open set in (Y, S) such that $f^{-1}(\lambda) \neq 1$. Since $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from the fuzzy hyperconnected space (X, T) into (Y, S), by Proposition 3.15, $int[f^{-1}(\lambda)] = 0$ in (X, T). Thus $f : (X, T) \to (Y, S)$ is not an somewhat fuzzy continuous function from (X, T) into (Y, S).

(2) From (1), $int[f^{-1}(\lambda)] = 0$ in (X, T) for an non-zero fuzzy open set in (Y, S). Thus $int[f^{-1}(\lambda)] \neq f^{-1}(\lambda)$. So $f : (X, T) \to (Y, S)$ is not an fuzzy continuous function from (X, T) into (Y, S). □

Proposition 3.17. *If $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from the fuzzy topological space (X, T) into the fuzzy topological space (Y, S) and λ is an non-zero fuzzy open set in (Y, S) such that $f^{-1}(\lambda) \neq 1$, then $f^{-1}(1 - \lambda)$ is an fuzzy somewhere dense set in (X, T).*

Proof. Let λ be an non-zero fuzzy open set in (Y, S) such that $f^{-1}(\lambda) \neq 1$. Since $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from the fuzzy topological space (X, T) into (Y, S), by Proposition 3.10, $f^{-1}(\lambda)$ is an fuzzy cs dense set in (X, T). Then $1 - f^{-1}(\lambda)$ is an fuzzy somewhere dense set in (X, T). Thus $f^{-1}(1 - \lambda)$ is an fuzzy somewhere dense set in (X, T). □

Proposition 3.18. *If $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from the fuzzy topological space (X, T) into the fuzzy strongly irresolvable space (Y, S) and λ is an non-zero fuzzy open set in (Y, S) such that $f^{-1}(\lambda) \neq 1$, then $f^{-1}(\lambda)$ is not an fuzzy dense set in (X, T).*

Proof. Let λ be an non-zero fuzzy open set in (Y, S) such that $f^{-1}(\lambda) \neq 1$. Since $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from the fuzzy topological space (X, T) into (Y, S), by Proposition 3.17, $f^{-1}(1 - \lambda)$ is an fuzzy somewhere dense set in (X, T). Since (X, T) is the fuzzy strongly irresolvable space, by Theorem 2.19, $int[f^{-1}(1 - \lambda)] \neq 0$ in (X, T). Then $int[1 - f^{-1}(\lambda)] \neq 0$. Thus by Lemma 2.2, $1 - cl[f^{-1}(\lambda)] \neq 0$. So $cl[f^{-1}(\lambda)] \neq 1$. Hence $f^{-1}(\lambda)$ is not an fuzzy dense set in (X, T). □

Proposition 3.19. *If $f : (X,T) \to (Y,S)$ is the A-somewhat fuzzy continuous function from the fuzzy topological space (X,T) into the fuzzy nodec space (Y,S) and λ is an non-zero fuzzy nowhere dense set in (Y,S) such that $f^{-1}(\lambda) \neq 1$, then there exists an non-zero fuzzy closed set δ in (X,T) such that $\delta \leq f^{-1}(\lambda)$.*

Proof. Let λ be an non-zero fuzzy nowhere dense set in (Y,S) such that $f^{-1}(\lambda) \neq 1$. Since (Y,S) is an fuzzy nodec space, the fuzzy nowhere dense set λ is an fuzzy closed set in (Y,S). Since $f : (X,T) \to (Y,S)$ is the A-somewhat fuzzy continuous function from (X,T) into (Y,S), there exists an non-zero fuzzy closed set δ in (X,T) such that $\delta \leq f^{-1}(\lambda)$. □

Proposition 3.20. *If $f : (X,T) \to (Y,S)$ is the A-somewhat fuzzy continuous function from the fuzzy topological space (X,T) into the fuzzy nodec space (Y,S) and λ is an fuzzy first category set in (Y,S) such that $f^{-1}(\lambda) \neq 1$, then there exists an non-zero fuzzy F_σ-set δ in (X,T) such that $\delta \leq f^{-1}(\lambda)$.*

Proof. Let λ be an fuzzy first category set in (Y,S) such that $f^{-1}(\lambda) \neq 1$. Then $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (Y,S). Clearly, $f^{-1}(\lambda_i) \neq 1$, [for, since $\lambda_i \leq \lambda$, $f^{-1}(\lambda_i) \leq f^{-1}(\lambda)$ and $f^{-1}(\lambda_i) = 1$, will imply that $f^{-1}(\lambda) = 1$, a contradiction]. Since $f : (X,T) \to (Y,S)$ is the A-somewhat fuzzy continuous function from (X,T) into (Y,S) for the fuzzy nowhere dense sets (λ_i)'s in (Y,S), by Proposition 3.19, there exist non-zero fuzzy closed sets (δ_i)'s in (X,T) such that $\delta_i \leq f^{-1}(\lambda_i)$. Thus $\vee_{i=1}^{\infty}(\delta_i) \leq \vee_{i=1}^{\infty}[f^{-1}(\lambda_i)]$ in (X,T). This implies that $\vee_{i=1}^{\infty}(\delta_i) \leq f^{-1}[\vee_{i=1}^{\infty}(\lambda_i)]$. So $\vee_{i=1}^{\infty}(\delta_i) \leq f^{-1}(\lambda)$. Let $\delta = \vee_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy closed sets in (X,T). Then δ is an fuzzy F_σ-set in (X,T). Thus for the fuzzy first category set λ in (Y,S) such that $f^{-1}(\lambda) \neq 1$, there exists an non-zero fuzzy F_σ-set δ in (X,T) such that $\delta \leq f^{-1}(\lambda)$. □

Proposition 3.21. *If $f : (X,T) \to (Y,S)$ is the A-somewhat fuzzy continuous function from the fuzzy hyperconnected and fuzzy open hereditarily irresolvable space (X,T) into the fuzzy topological space (Y,S) and λ is an non-zero fuzzy open set in (Y,S) such that $f^{-1}(\lambda) \neq 1$, $f^{-1}(\lambda)$ is an fuzzy nowhere dense set in (X,T).*

Proof. Let λ be an non-zero fuzzy open set in (Y,S) such that $f^{-1}(\lambda) \neq 1$. Since $f : (X,T) \to (Y,S)$ is the A-somewhat fuzzy continuous function from the fuzzy hyperconnected space (X,T) into (Y,S), by Proposition 3.15, $int[f^{-1}(\lambda)] = 0$, in (X,T). Also since (X,T) is an fuzzy open hereditarily irresolvable space, $int[f^{-1}(\lambda)] = 0$ in (X,T). Then by Theorem 2.21, $intcl[f^{-1}(\lambda)] = 0$. Thus $f^{-1}(\lambda)$ is an fuzzy nowhere dense set in (X,T). □

Proposition 3.22. *If $f : (X,T) \to (Y,S)$ is the A-somewhat fuzzy continuous function from the fuzzy hyper connected and fuzzy open hereditarily irresolvable space (X,T) into the fuzzy topological space (Y,S) and λ is an fuzzy G_δ-set in (Y,S) such that $f^{-1}(\lambda) \neq 1$, then $f^{-1}(\lambda)$ is an fuzzy nowhere dense set in (X,T).*

Proof. Let λ be an fuzzy G_δ-set in (Y,S) such that $f^{-1}(\lambda) \neq 1$. Then $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy open sets in (Y,S). Since $f : (X,T) \to (Y,S)$ is the A-somewhat fuzzy continuous function from the fuzzy hyperconnected and fuzzy open hereditarily irresolvable space (X,T) into (Y,S), by Proposition 3.21, $[f^{-1}(\lambda_i)]$'s are fuzzy nowhere dense sets in (X,T). Now $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$ implies that $f^{-1}(\lambda) =$

$f^{-1}[\wedge_{i=1}^{\infty}(\lambda_i)] = \wedge_{i=1}^{\infty}[f^{-1}(\lambda_i)]$. Thus $f^{-1}(\lambda)$ is an fuzzy nowhere dense set in (X, T). □

Proposition 3.23. *If $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from the fuzzy topological space (X, T) into the fuzzy topological space (Y, S) and λ is an non-zero fuzzy open set in (Y, S) such that $f^{-1}(\lambda) \neq 1$, then there exists an fuzzy closed set μ in (X, T) such that $int[f^{-1}(\lambda)] \leq \mu$.*

Proof. Let λ be an non-zero fuzzy open set in (Y, S) such that $f^{-1}(\lambda) \neq 1$. Since $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from (X, T) into (Y, S), by Proposition 3.10, $f^{-1}(\lambda)$ is an fuzzy cs dense set in (X, T). By theorem 2.18, there exists an fuzzy closed set μ in (X, T) such that $int[f^{-1}(\lambda)] \leq \mu$. □

Proposition 3.24. *If $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from the fuzzy hyperconnected and fuzzy strongly irresolvable space (X, T) into the fuzzy topological space (Y, S) and μ is an non-zero fuzzy closed set in (Y, S) such that $f^{-1}(\mu) \neq 0$ and $f^{-1}(\mu) \leq \delta$, where δ is an fuzzy set defined on X, then $1 - \delta$ is an fuzzy nowhere dense set in (X, T).*

Proof. Let μ be an non-zero fuzzy closed set in (Y, S) such that $f^{-1}(\mu) \neq 0$. Since $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from the fuzzy hyperconnected space (X, T) into (Y, S), by Proposition 3.12 (1), $cl[f^{-1}(\mu)] = 1$ in (X, T). Then $f^{-1}(\mu)$ is an fuzzy dense set in (X, T). Thus by the hypothesis, $f^{-1}(\mu) \leq \delta$, where δ is an fuzzy set defined on X. Since (X, T) is an fuzzy strongly irresolvable space, $f^{-1}(\mu) \leq \delta$ and $f^{-1}(\mu)$ is an fuzzy dense set in (X, T). So by Theorem 2.23, that $1 - \delta$ is an fuzzy nowhere dense set in (X, T). □

The following proposition gives the conditions by means of A-somewhat fuzzy continuous functions for the fuzzy hyperconnected and fuzzy strongly irresolvable spaces to become fuzzy Baire spaces.

Proposition 3.25. *If $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from the fuzzy hyper connected and fuzzy strongly irresolvable space (X, T) into the fuzzy topological space (Y, S) and (μ_i)'s $(i = 1$ to $\infty)$ are non-zero fuzzy closed sets in (Y, S) such that $f^{-1}(\mu_i) \neq 0$ and $cl[\wedge_{i=1}^{\infty}\left(f^{-1}(\mu_i)\right)] = 1$, then (X, T) is the fuzzy Baire space.*

Proof. Let μ_i's $(i = 1$ to $\infty)$ be fuzzy closed sets in (Y, S) such that $f^{-1}(\mu_i) \neq 0$. Since $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from the fuzzy hyperconnected and fuzzy strongly irresolvable space (X, T) into (Y, S), by Proposition 3.24, $(1 - \delta_i)$'s are fuzzy nowhere dense sets in (X, T), where (δ_i)'s are the fuzzy sets defined on X such that $f^{-1}(\mu_i) \leq \delta_i$. Now the hypothesis, $cl[\wedge_{i=1}^{\infty}\left(f^{-1}(\mu_i)\right)] = 1$. Then $1 - cl[\wedge_{i=1}^{\infty}\left(f^{-1}(\mu_i)\right)] = 0$. Thus $int[1 - \wedge_{i=1}^{\infty}\left(f^{-1}(\mu_i)\right)] = 0$. So $int[\vee_{i=1}^{\infty}\left(f^{-1}(1 - \mu_i)\right)] = 0$. Since $int[\vee_{i=1}^{\infty}(1 - \delta_i)] \leq int[\vee_{i=1}^{\infty}\left(f^{-1}(1 - \mu_i)\right)]$, $int[\vee_{i=1}^{\infty}(1 - \delta_i)] = 0$, where $(1 - \delta_i)$'s are fuzzy nowhere dense sets in (X, T). Hence (X, T) is the fuzzy Baire space. □

Proposition 3.26. *If $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from the fuzzy hyperconnected and fuzzy strongly irresolvable space (X, T) into the fuzzy topological space (Y, S) and μ_i's $(i = 1$ to $\infty)$ are non-zero fuzzy* *closed sets in* (Y, S) *such that* $f^{-1}(\mu_i) \neq 0$ *and* $cl[\wedge_{i=1}^{\infty}\left(f^{-1}(\mu_i)\right)] = 1$, *then* (X, T) *is the fuzzy second category space.*

Proof. The proof follows from Proposition 3.25 and Theorem 2.25. □

Proposition 3.27. *If the function* $f : (X, T) \to (Y, S)$ *is the A-somewhat fuzzy continuous function from the fuzzy topological space* (X, T) *into the fuzzy topological space* (Y, S) *and* λ_i*'s* $(i = 1 \text{ to } \infty)$ *are non-zero fuzzy open sets in* (Y, S) *such that* $f^{-1}(\lambda_i) \neq 0$, *then there exists an fuzzy open set* λ *and fuzzy* F_σ*-set* η *in* (X, T) *such that* $\lambda \leq \eta$, *where* $\lambda = \vee_{i=1}^{\infty}[int\left(f^{-1}(\lambda_i)\right)]$ *and* $\eta = \vee_{i=1}^{\infty}(\eta_i)$, $1 - \eta_i \in T$.

Proof. Let λ_i's $(i = 1 \text{ to } \infty)$ be fuzzy open sets in (Y, S) such that $f^{-1}(\lambda_i) \neq 0$. Since $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from (X, T) into (Y, S), by Proposition 3.10, $[f^{-1}(\lambda_i)]$'s are fuzzy cs dense sets in (X, T). Then by Theorem 2.24, for the fuzzy cs dense sets $[f^{-1}(\lambda_i)]$'s in (X, T), there exists an fuzzy open set λ and fuzzy F_σ-set η in (X, T) such that $\lambda \leq \eta$, where $\lambda = \vee_{i=1}^{\infty}[int\left(f^{-1}(\lambda_i)\right)]$ and $\eta = \vee_{i=1}^{\infty}(\eta_i)$, $1 - \eta_i \in T$. □

Proposition 3.28. *If the function* $f : (X, T) \to (Y, S)$ *is A-somewhat fuzzy continuous function from the fuzzy topological space* (X, T) *into the fuzzy topological space* (Y, S) *and* μ *is an non-zero fuzzy closed set in* (Y, S) *such that* $f^{-1}(\mu) \neq 0$, *then there exists an fuzzy cs dense set* δ *in* (X, T) *such that* $\delta \leq f^{-1}(\mu)$.

Proof. Let μ be an non-zero fuzzy closed set in (Y, S) such that $f^{-1}(\mu) \neq 0$. Since $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from (X, T) into (Y, S), there exists an non-zero fuzzy closed set δ in (X, T) such that $\delta \leq f^{-1}(\mu)$. Then by Theorem 2.26, the fuzzy closed set δ is an fuzzy cs dense set in (X, T). □

The following proposition gives the conditions by means of A-somewhat fuzzy continuous functions for the fuzzy hyperconnected to become fuzzy almost irresolvable spaces.

Proposition 3.29. *If the function* $f : (X, T) \to (Y, S)$ *is A-somewhat fuzzy continuous function from the fuzzy hyperconnected space* (X, T) *into the fuzzy topological space* (Y, S) *and* μ_i*'s* $(i = 1 \text{ to } \infty)$ *are non-zero fuzzy closed sets in* (Y, S) *such that* $f^{-1}(\mu_i) \neq 0$ *and* $\vee_{i=1}^{\infty}[f^{-1}(\mu_i)] \neq 1$, *then* (X, T) *is the fuzzy almost irresolvable space.*

Proof. Let μ_i's $(i = 1 \text{ to } \infty)$ be fuzzy closed sets in (Y, S) such that $f^{-1}(\mu_i) \neq 0$. Since $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from (X, T) into (Y, S), by Proposition 3.28, there exist fuzzy cs dense sets (δ_i)'s in (X, T) such that $\delta_i \leq f^{-1}(\mu_i)$. This implies that $\vee_{i=1}^{\infty}(\delta_i) \leq \vee_{i=1}^{\infty}[f^{-1}(\mu_i)]$. Then $\vee_{i=1}^{\infty}(\delta_i) \neq 1$, in (X, T). [for, if $\vee_{i=1}^{\infty}(\delta_i) = 1$, will imply that $\vee_{i=1}^{\infty}[f^{-1}(\mu_i)] = 1$, a contradiction]. Thus $\vee_{i=1}^{\infty}(\delta_i) \neq 1$, where (δ_i)'s are fuzzy cs dense sets in the fuzzy hyperconnected space (X, T). So by Theorem 2.27, (X, T) is the fuzzy almost irresolvable space. □

Proposition 3.30. *If* $f : (X, T) \to (Y, S)$ *is the A-somewhat fuzzy continuous function from the fuzzy topological space* (X, T) *into the fuzzy topological space* (Y, S) *and* λ *is an non-zero fuzzy open set in* (Y, S) *such that* $f^{-1}(\lambda) \neq 0$, *then there exists an fuzzy regular open* δ *in* (X, T) *such that* $int[f^{-1}(\lambda)] \leq \delta$.

Proof. Let λ be an non-zero fuzzy open set in (Y, S) such that $f^{-1}(\lambda) \neq 0$. Since the function $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from (X, T) into (Y, S), by Proposition 3.10, $f^{-1}(\lambda)$ is an fuzzy cs dense set in (X, T). Then $1 - f^{-1}(\lambda)$ is an fuzzy somewhere dense set in (X, T). Thus by Theorem 2.28, there exists an fuzzy regular closed set η in (X, T) such that $\eta \leq cl[1 - f^{-1}(\lambda)]$. This implies that $\eta \leq 1 - int[f^{-1}(\lambda)]$ and then $int[f^{-1}(\lambda)] \leq 1 - \eta$. Let $\delta = 1 - \eta$. So δ is an fuzzy regular open set in (X, T). Hence for the non-zero fuzzy open set λ in (Y, S) such that $f^{-1}(\lambda) \neq 0$, there exists an fuzzy regular open δ in (X, T) such that $int[f^{-1}(\lambda)] \leq \delta$ □

Proposition 3.31. *If the function $f : (X, T) \to (Y, S)$ is A-somewhat fuzzy continuous function from the fuzzy perfectly disconnected space (X, T) into the fuzzy topological space (Y, S) and μ_i's ($i = 1$ to ∞) are non-zero fuzzy closed sets in (Y, S) such that $f^{-1}(\mu_i) \neq 0$, then there exists an fuzzy G_δ-set η in (X, T) such that $\eta \leq \wedge_{i=1}^{\infty}[cl\left(f^{-1}(\mu_i)\right)]$.*

Proof. Let μ_i's ($i = 1$ to ∞) be fuzzy closed sets in (Y, S) such that $f^{-1}(\mu_i) \neq 0$. Since $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from (X, T) into (Y, S), by Proposition 3.8, $[f^{-1}(\mu_i)]$'s are fuzzy somewhere dense sets in (X, T). Since (X, T) is an fuzzy perfectly disconnected space, by Theorem 2.30, there exists an fuzzy G_δ-set η in (X, T) such that $\eta \leq \wedge_{i=1}^{\infty}[cl\left(f^{-1}(\mu_i)\right)]$. □

Proposition 3.32. *If the function $f : (X, T) \to (Y, S)$ is A-somewhat fuzzy continuous function from the fuzzy topological space (X, T) into the fuzzy perfectly disconnected space (Y, S) and μ is an fuzzy set defined on Y such that $f^{-1}(\mu) \neq 0$ in (Y, S), then there exists an non-zero fuzzy closed set δ in (X, T) such that $\delta \leq f^{-1}(\mu)$.*

Proof. Let μ be an fuzzy set defined on Y such that $f^{-1}(\mu) \neq 0$ in (Y, S). Since (Y, S) is an fuzzy perfectly disconnected space, by Theorem 2.29, $int(\mu)$ is an fuzzy closed set in (X, T). Since $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from (X, T) into (Y, S), there exists an non-zero fuzzy closed set δ in (X, T) such that $\delta \leq f^{-1}[int(\mu)]$. Since $f^{-1}[int(\mu)] \leq f^{-1}(\mu)$ for the fuzzy set μ defined on Y, there exists an non-zero fuzzy closed set δ in (X, T) such that $\delta \leq f^{-1}(\mu)$. □

Proposition 3.33. *If the function $f : (X, T) \to (Y, S)$ is A-somewhat fuzzy continuous function from the fuzzy topological space (X, T) into the fuzzy perfectly disconnected space (Y, S) and λ is an fuzzy open set such that $f^{-1}(\lambda) \neq 0$ in (Y, S), then there exists an non-zero fuzzy closed set δ in (X, T) such that $\delta \leq f^{-1}[cl(\mu)]$.*

Proof. Let λ be an non-zero fuzzy open set in (Y, S) such that $f^{-1}(\lambda) \neq 0$. Since (Y, S) is an fuzzy perfectly disconnected space, by Theorem 2.31, $intcl(\lambda)$ is an fuzzy closed set in (Y, S). Since $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from (X, T) into (Y, S), there exists an non-zero fuzzy closed set δ in (X, T) such that $\delta \leq f^{-1}[intcl(\lambda)]$. Since $f^{-1}[intcl(\lambda)] \leq f^{-1}[cl(\lambda)]$ in (X, T). Then for the fuzzy open set λ in (Y, S), there exists an non-zero fuzzy closed set δ in (X, T) such that $\delta \leq f^{-1}[cl(\mu)]$ □

The following proposition gives a condition for fuzzy pre-continuous functions to become A-somewhat fuzzy continuous functions.

Proposition 3.34. *If $f : (X, T) \to (Y, S)$ is an fuzzy pre-continuous function from the fuzzy topological space (X, T) in which there is no fuzzy dense set into the fuzzy topological space (Y, S), then $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from (X, T) into (Y, S).*

Proof. Let λ be an non-zero fuzzy open set in (Y, S) such that $f^{-1}(\lambda) \neq 1$. Since the function $f : (X, T) \to (Y, S)$ is an fuzzy pre-continuous function from (X, T) into (Y, S), $f^{-1}(\lambda)$ is an fuzzy pre-open set in (X, T). This implies that $f^{-1}(\lambda) \leq intcl[f^{-1}(\lambda)]$. Since there is no fuzzy dense set in (X, T), $cl[f^{-1}(\lambda)] \neq 1$ and hence $int\{cl[f^{-1}(\lambda)]\} \neq 1$. Let $\eta = intcl[f^{-1}(\lambda)]$ and then $\eta \neq 1$. Then for the fuzzy open set λ in (Y, S) such that $f^{-1}(\lambda) \neq 1$, there exists an fuzzy open set $\eta (\neq 1)$ in (X, T) such that $f^{-1}(\lambda) \leq \eta$. Thus by Proposition 3.9, $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from (X, T) into (Y, S). □

The following proposition shows that if the function is the A-somewhat fuzzy continuous function from the fuzzy topological space (X, T) into the fuzzy submaximal space (Y, S), then (X, T) is not an fuzzy hyperconnected space.

Proposition 3.35. *If the function $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from the fuzzy topological space (X, T) into the fuzzy submaximal space (Y, S), then (X, T) is not an fuzzy hyperconnected space.*

Proof. Let λ be an fuzzy dense set in (Y, S) such that $f^{-1}(\lambda) \neq 1$. Since (Y, S) is an fuzzy submaximal space, λ is an fuzzy open set in (Y, S). Since $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from (X, T) into (Y, S). Then by Proposition 3.10, $f^{-1}(\lambda)$ is an fuzzy cs dense set in (X, T). By Theorem 2.18, there exists an fuzzy closed set μ in (X, T) such that $int(f^{-1}(\lambda)) \leq \mu$. Thus $clint(f^{-1}(\lambda)) \leq cl(\mu)$ in (X, T). This implies that $clint(f^{-1}(\lambda)) \leq \mu$, in (X, T) [since μ is fuzzy closed, $cl(\mu) = \mu$]. So $clint(f^{-1}(\lambda)) \neq 1$. Hence $int(f^{-1}(\lambda))$ is not an fuzzy dense set in (X, T). Therefore for the fuzzy open set $int(f^{-1}(\lambda))$ in (X, T), $int(f^{-1}(\lambda))$ is not an fuzzy dense set in (X, T) implies that (X, T) is not an fuzzy hyperconnected space. □

Proposition 3.36. *If the function $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from the fuzzy topological space (X, T) into the fuzzy fuzzy nodec and fuzzy open hereditarily irresolvable space (Y, S) and λ is an fuzzy dense set in (Y, S), then there exists an fuzzy open set η in (X, T) such that $f^{-1}(\lambda) \leq \eta$.*

Proof. Let λ be an fuzzy dense set in (Y, S). Then $cl(\lambda) = 1$ and this implies that $1 - cl(\lambda) = 0$ and thus $int(1 - \lambda) = 0$ in (Y, S). Since (Y, S) is fuzzy open hereditarily irresolvable space, $int(1 - \lambda) = 0$. So $intcl(1 - \lambda) = 0$. Hence $1 - \lambda$ is an fuzzy nowhere dense set in (Y, S). Since (Y, S) is an fuzzy nodec space, the fuzzy nowhere dense set $1 - \lambda$ is an fuzzy closed set in (Y, S). Since $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from (X, T) into (Y, S), there exists an non-zero fuzzy closed set δ in (X, T) such that $\delta \leq f^{-1}(1 - \lambda)$. Therefore $\delta \leq 1 - f^{-1}(\lambda)$. This implies that $f^{-1}(\lambda) \leq 1 - \delta$. Let $\eta = 1 - \delta$. Then for the fuzzy dense set λ in (Y, S), there exists an non-zero fuzzy open set η in (X, T) such that $f^{-1}(\lambda) \leq \eta$. □

Proposition 3.37. *If $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from the fuzzy hyperconnected space (X, T) into the fuzzy topological space*

(Y, S) and μ is an non-zero fuzzy closed set in (Y, S) such that $f^{-1}(\mu) \neq 0$, then there exists a fuzzy resolvable set δ in (X, T) such that $\delta \leq cl[f^{-1}(\mu)]$.

Proof. Let μ be an non-zero fuzzy closed set in (Y, S) such that $f^{-1}(\mu) \neq 0$. Since $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from the fuzzy hyperconnected space (X, T) into (Y, S), by Proposition 3.13, $f^{-1}(\mu)$ is an fuzzy β-open set in (X, T). Then by Theorem 2.32, there exists an fuzzy resolvable set δ in (X, T) such that $\delta \leq cl[f^{-1}(\mu)]$. □

The following proposition gives the conditions by means of A-somewhat fuzzy continuous functions for the fuzzy hyperconnected spaces to become fuzzy Baire spaces.

Proposition 3.38. *Let $f : (X, T) \to (Y, S)$ be the A-somewhat fuzzy continuous function from the fuzzy hyperconnected space (X, T) into the fuzzy topological space (Y, S) and μ_i's ($i = 1$ to ∞) are non-zero fuzzy closed sets in (Y, S) such that $f^{-1}(\mu_i) \neq 0$. If $int\{\vee_{i=1}^{\infty}[cl\left(f^{-1}(\mu_i)\right)]\} = 0$, in (X, T), then (X, T) is an fuzzy Baire space.*

Proof. Let μ_i's ($i = 1$ to ∞) be fuzzy closed sets in (Y, S) such that $f^{-1}(\mu_i) \neq 0$. Since $f : (X, T) \to (Y, S)$ is the A-somewhat fuzzy continuous function from (X, T) into (Y, S), by Proposition 3.37, there exist fuzzy resolvable sets (δ_i)'s in (X, T) such that $\delta_i \leq cl[f^{-1}(\mu_i)]$. This implies that $int[\vee_{i=1}^{\infty}(\delta_i)] \leq int\{\vee_{i=1}^{\infty}\left(cl[f^{-1}(\mu_i)]\right)\}$ in (X, T). If $int\{\vee_{i=1}^{\infty}\left(cl[f^{-1}(\mu_i)]\right)\} = 0$, implies that $int[\vee_{i=1}^{\infty}(\delta_i)] = 0$, where (δ_i)'s are fuzzy resolvable sets in (X, T), then by Theorem 2.33, (X, T) is an fuzzy Baire space □

4. Conclusion

In this paper, a new form of somewhat fuzzy continuity, called A-somewhat fuzzy continuity between fuzzy topological spaces is introduced and studied. It is shown that A-somewhat fuzzy continuity is independent of somewhat fuzzy continuity. Several characterizations of these functions are established. The conditions for fuzzy α-continuous functions and fuzzy pre-continuous functions to become A- somewhat fuzzy continuous functions, are obtained. It is shown that A-somewhat fuzzy continuous function from the fuzzy hyperconnected space into another fuzzy topological space is not an somewhat fuzzy continuous function. The conditions by means of A-somewhat fuzzy continuous functions for the fuzzy hyperconnected and fuzzy strongly irresolvable spaces to become fuzzy Baire spaces, are also obtained. The conditions by means A-somewhat fuzzy continuous functions and fuzzy resolvable sets for the fuzzy hyperconnected spaces to become fuzzy Baire spaces, are also obtained. It is shown that if the function is the A-somewhat fuzzy continuous function from the fuzzy topological space (X, T) into the fuzzy submaximal space (Y, S), then (X, T) is not an fuzzy hyperconnected space.

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