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Abstract. The domination polynomial of a fuzzy graph is the generating function of its dominating sets. In this paper, domination polynomial in fuzzy graphs using strong arcs is introduced. The domination polynomial of different classes of fuzzy graphs is obtained. It is obtained some properties of the domination polynomial and its coefficients.

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1. Introduction

In recent years there has been growing interest in fuzzy graph polynomials. Fuzzy graph polynomials encode information about the fuzzy graphs in a compact way in their evaulations, coeffcients, degree and roots. Therefore, efficient computation of fuzzy graph polynomials has received considerable attention in the literature. Fuzzy graph polynomials are powerful and well-developed tools to express fuzzy graph parameters. There are numerous polynomials associated with fuzzy graphs. By studying these polynomials one can obtain some properties of a fuzzy graph. The domination polynomial is the ordinary generating function of several dominating sets in a fuzzy graph. Fuzzy graphs were introduced by Rosenfeld [1]. Rosenfeld has described the fuzzy analogue of several graph theoretic concepts like paths, cycles, trees and connectedness and established some of their properties [1]. Bhutani and Rosenfeld have introduced the concept of strong arcs [2]. The concept of domination in graphs was introduced by Ore and Berge in 1962 ,the domination number and independent domination number are introduced by Cockayne and Hedetniemi [3]. A.Somasundaram and S.Somasundaram discussed domination in fuzzy graphs [4, 5]. They defined domination using effective edges in fuzzy graph. Nagoor gani and Chandrasekaran [6] introduced the concept of domination using strong arcs.

According to Nagoor gani a node v in a fuzzy graph G is said to strongly dominate itself and each of its strong neighbors, that is, v strongly dominates the nodes in $N_s[v]$. A set D of nodes of G is a strong dominating set of G if every node of $V(G)-D$ is a strong neighbor of some node in D. They defined a minimum strong dominating set in a fuzzy graph G as a strong dominating set with minimum number of nodes and it is denoted by $\gamma_s(G)$ or simply γ_s. More complicated structures of fuzzy graphs such as bipolar fuzzy graphs , intutionistic fuzzy graphs and domination in these fuzzy graphs were studied by Akram, Nagoorgani, palnivel, and sivasankar [7, 8, 9, 10]. In this paper, domination polynomial of fuzzy graphs using strong arcs is introduced. The domination polynomial of different classes of fuzzy graphs is obtained. It is obtained some properties of the domination polynomial and its coefficients.

2. Preliminaries

It is quite known that graphs are simply models of relations. A graph is a convenient way of representing information involving relationship between objects. The objects are represented by vertices and relations, by edges. When there is vagueness in the description of the objects or in its relationships or in both, it is natural that we need to design a 'fuzzy graph model'. We summarize briefly some basic definitions in fuzzy graphs which are presented in [1, 2, 4, 6, 11, 12, 13, 14, 15].

A *fuzzy graph* is denoted by $G : (V, \sigma, \mu)$, where V is a node set, σ and μ are mappings defined as $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$, where σ and μ represent the membership values of a node and an arc respectively. For any fuzzy graph, $\mu(x, y) \leq min\{\sigma(x), \sigma(y)\}$. We consider fuzzy graph G with no loops and assume that V is finite and nonempty, μ is reflexive, i.e.,$\mu(x, x) = \sigma(x)$ for all x and symmetric, i.e., $\mu(x, y) = \mu(y, x)$ for all (x, y). In all the examples σ is chosen suitably. Also, we denote the underlying crisp graph by $G^* : (\sigma^*, \mu^*)$ where $\sigma^* = \{u \in V : \sigma(u) > 0\}$ and $\mu^* = \{(u, v) \in V \times V : \mu(u, v) > 0\}$. Throughout this paper, we assume that $\sigma^* = V$. The fuzzy graph $H : (\tau, \nu)$ is said to be a *partial fuzzy subgraph* of $G : (V, \sigma, \mu)$, if $\nu \subseteq \mu$ and $\tau \subseteq \sigma$. In particular, we call $H : (\tau, \nu)$ a *fuzzy subgraph* of $G : (V, \sigma, \mu)$, if $\tau(u) = \sigma(u)$ for all $u \in \tau^*$ and $\nu(u, v) = \mu(u, v)$ for all $(u, v) \in \nu^*$. A fuzzy graph $G : (V, \sigma, \mu)$ is called *trivial*, if $|\sigma^*| = 1$. Two nodes u and v in a fuzzy graph G are said to be *adjacent* (*neighbors*), if $\mu(u, v) > 0$. The set of all neighbors of u is denoted by $N(u)$.

An arc (u, v) of a fuzzy graph $G : (V, \sigma, \mu)$ with $\mu(u, v) > 0$ is called a *weakest arc* of G, if (u, v) is an arc with minimum $\mu(u, v)$.

A *path* P of length n is a sequence of distinct nodes $u_0, u_1, ..., u_n$ such that $\mu(u_{i-1}, u_i) > 0, i = 1, 2, ..., n$ and the degree of membership of a weakest arc is defined as its strength. If $u_0 = u_n$ and $n \geq 3$, then P is called a *cycle*and P is called a *fuzzy cycle*, if it contains more than one weakest arc. The *strength of a cycle* is the strength of the weakest arc in it. The *strength of connectedness* between two nodes x and y is defined as the maximum of the strengths of all paths between x and y and is denoted by $CONN_G(x, y)$.

A fuzzy graph $G : (V, \sigma, \mu)$ is *connected*, if for every x, y in $\sigma^*, CONN_G(x, y) > 0$.

An arc (u, v) of a fuzzy graph is called an *effective arc*, if $\mu(u, v) = \sigma(u) \wedge \sigma(v)$. Then u and v are called effective neighbors. The set of all effective neighbors of u is called *effective neighborhood* of u and is denoted by $EN(u)$.

A fuzzy graph $G : (V, \sigma, \mu)$ is said to be *complete*, if $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ for all $u, v \in \sigma^*$.

The *order* p and *size* q of a fuzzy graph $G : (V, \sigma, \mu)$ are defined to be $p = \sum_{x \in V} \sigma(x)$ and $q = \sum_{(x,y) \in V \times V} \mu(x, y)$.

Let $G : (V, \sigma, \mu)$ be a fuzzy graph and $S \subseteq V$. Then the *scalar cardinality* of S is defined to be $\sum_{v \in S} \sigma(v)$ and it is denoted by $|S|_s$. Let p denotes the scalar cardinality of V, also called the order of G.

The *complement* of a fuzzy graph $G : (V, \sigma, \mu)$, denoted by $\overline{G}$ is defined to be $\overline{G} = (V, \sigma, \overline{\mu})$ where $\overline{\mu}(x, y) = \sigma(x) \wedge \sigma(y) - \mu(x, y)$ for all $x, y \in V$ [16].

An *arc* of a fuzzy graph $G : (V, \sigma, \mu)$ is called *strong*, if its weight is at least as great as the strength of connectedness of its end nodes when it is deleted. A fuzzy graph G is called a *strong fuzzy graph* if each arc in G is a strong arc. Depending on $CONN_G(x, y)$ of an arc (x, y) in a fuzzy graph G, Mathew and Sunitha [15] defined three different types of arcs. Note that $CONN_{G-(x,y)}(x, y)$ is the the strength of connectedness between x and y in the fuzzy graph obtained from G by deleting the arc (x, y). An arc (x, y) in G is *α-strong*, if $\mu(x, y) > CONN_{G-(x,y)}(x, y)$. An arc (x, y) in G is *β-strong*, if $\mu(x, y) = CONN_{G-(x,y)}(x, y)$. An arc (x, y) in G is *δ-arc*, if $\mu(x, y) < CONN_{G-(x,y)}(x, y)$. Thus an arc (x, y) is a strong arc if it is either α-strong or β-strong. Also y is called *strong neighbor* of x, if arc (x,y) is strong. The set of all strong neighbors of x is called the *strong neighborhood* of x and is denoted by $N_s(x)$. The *closed strong neighborhood* $N_s[x]$ is defined as $N_s[x] = N_s(x) \cup \{x\}$. A path P is called *strong path*, if P contains only strong arcs.

A fuzzy graph $G : (V, \sigma, \mu)$ is said to be *bipartite* [4], if the vertex set V can be partitioned into two non empty sets V_1 and V_2 such that $\mu(v_1, v_2) = 0$ if $v_1, v_2 \in V_1$ or $v_1, v_2 \in V_2$. Further if $\mu(u, v) = \sigma(u) \bigwedge \sigma(v)$ for all $u \in V_1$ and $v \in V_2$ then G is called a *complete bipartite graph* and is denoted by K_{σ_1,σ_2}, where σ_1 and σ_2 are respectively the restrictions of σ to V_1 and V_2.

A *node* u is said to be *isolated*, if $\mu(u, v) = 0$ for all $v \neq u$.

3. Domination polynomials in fuzzy graphs

In this section, domination polynomials in fuzzy graphs is introduced. Domination polynomials in fuzzy graphs using strong arcs give rise to a clear cut representation of the fuzzy graphs. First let us recall the domination concept in fuzzy graphs using strong arcs as follows. The concept of strong domination in fuzzy graphs has applications to several fields. Nagoor gani and Chandrasekaran [4] introduced the concept of domination using strong arcs. According to Nagoor gani a node v in a fuzzy graph G is said to strongly dominate itself and each of its strong neighbors, that is, v strongly dominates the nodes in $N_s[v]$. A set D of nodes of G is a strong dominating set of G if every node of $V(G) - D$ is a strong neighbor of some node in D. They defined a minimum strong dominating set in a fuzzy graph G as a strong dominating set with minimum number of nodes [6] and it is denoted by $\gamma_s(G)$ or simply γ_s.

Now let us define domination polynomials in fuzzy graphs using strong arcs as follows.

Definition 3.1. Let $D_s(G,m)$ be the family of strong dominating sets of a fuzzy graph G with cardinality m and let $d_s(G,m) = |D_s(G,m)|$. Then strong domination polynomial $D_s(G,x)$ of G is defined as:

$$D_s(G,x) = \sum_{m=\gamma_s(G)}^{|V(G)|} d_s(G,m)x^m,$$

where $\gamma_s(G)$ is the strong domination number of G.

Example 3.2. Consider a fuzzy cycle F having 5 nodes. All of its arcs are strong. Then F contains

only one strong dominating set having 5 nodes,
5 strong dominating sets having 4 nodes each,
5 strong dominating sets having 3 nodes each,
5 strong dominating sets having 2 nodes each.

Thus the strong domination polynomial of F is

$$D_s(F,x) = x^5 + 5x^4 + 5x^3 + 5x^2.$$

The path P on 4 nodes has one strong dominating set of cardinality 4, four strong dominating sets cardinalies 3 and 2. So its strong domination polynomial is

$$D_s(P,x) = x^4 + 4x^3 + 4x^2.$$

The strong domination polynomial of the complete fuzzy graph K_σ on n nodes is

$$D_s(K_\sigma,x) = (1+x)^n - 1.$$

Theorem 3.3. *If a fuzzy graph G consists of n components $G_1, G_2, G_3, \ldots\ldots, G_n$, then $D_s(G,x) = D_s(G_1,x)D_s(G_2,x) \ldots\ldots D_s(G_n,x)$.*

Proof. It is enough to prove the result for $n = 2$. For $i \geq \gamma_s(G)$, a strong dominating set of i nodes in G is obtained by choosing a strong dominating set of k nodes in G_1 (for some $k \in \gamma_s(G_1), \gamma_s(G_1)+1, \ldots\ldots\ldots |V(G_1|)$) and a strong dominating set of $i-k$ nodes in G_2. The number of possible ways of doing this over all $k = \gamma_s(G_1), \gamma_s(G_1)+1, \ldots\ldots\ldots |V(G_1)|$ is exactly the coefficient of x^i in $D_s(G_1,x)D_s(G_2,x)$. Then both sides of strong domination polynomial equation have the same coefficient. Thus they are identical polynomials. □

The following corollary for the empty graphs is an immediate consequence of Theorem 3.3.

Corollary 3.4. *Let $\bar{K_\sigma}$ be the empty fuzzy graph with n nodes. Then $D_s(\bar{K_\sigma},x) = x^n$.*

Proof. The empty fuzzy graph $\bar{K_\sigma}$ on one node contains only one strong dominating set of cardinality one. Then $D_s(\bar{K_\sigma},x) = x$ for the empty fuzzy graph $\bar{K_\sigma}$ on one node. Now $\bar{K_\sigma}$ with n nodes is a union of n empty fuzzy graphs $\bar{K_\sigma}$ components having one node. Thus by applying Theorem 3.3, the desired result is obtained. □

Next we find a formula for the strong domination polynomial of the join of two fuzzy graphs. First let us recall the definition of join of two fuzzy graphs.

Definition 3.5 ([13, 17]). Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs with $G_1^* : (V_1, E_1)$ and $G_2^* : (V_2, E_2)$ with $V_1 \cap V_2 = \phi$ and let $G^* = G_1^* \cup G_2^* = (V_1 \cup V_2, E_1 \cup E_2)$ be the union of G_1^* and G_2^*. Then the *union* of two fuzzy graphs G_1 and G_2 is a fuzzy graph $G : (\sigma_1 \cup \sigma_2, \mu_1 \cup \mu_2)$ defined by

$$(\sigma_1 \cup \sigma_2)(u) = \begin{cases} \sigma_1(u) & \text{if } u \in V_1 \setminus V_2 \\ \\ \sigma_2(u) & \text{if } u \in V_2 \setminus V_1, \end{cases}$$

and

$$(\mu_1 \cup \mu_2)(u, v) = \begin{cases} \mu_1(u, v) & \text{if } (u, v) \in E_1 \setminus E_2 \\ \\ \mu_2(u, v) & \text{if } (u, v) \in E_2 \setminus E_1. \end{cases}$$

Example 3.6. Union of any two fuzzy graphs is a new fuzzy graph with node set as the union of the nodes of the two fuzzy graphs and arc set as the union of the arcs of the two fuzzy graphs. Geometrically we cannot express the difference as well.

Definition 3.7 ([13, 17]). Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs with $G_1^* : (V_1, E_1)$ and $G_2^* : (V_2, E_2)$ with $V_1 \cap V_2 = \phi$ Consider the join $G^* = G_1^* + G_2^* = (V_1 \cup V_2, E_1 \cup E_2 \cup E^{'})$ of graphs, where $E^{'}$ is the set of all arcs joining the nodes of V_1 and V_2, where we assume that $V_1 \cap V_2 = \phi$. Then the *join* of two fuzzy graphs G_1 and G_2 is a fuzzy graph $G = G_1 + G_2 : (\sigma_1 + \sigma_2, \mu_1 + \mu_2)$ defined by

$$(\sigma_1 + \sigma_2)(u) = (\sigma_1 \cup \sigma_2)(u),\ u \in V_1 \cup V_2$$

and

$$(\mu_1 + \mu_2)(u, v) = \begin{cases} (\mu_1 \cup \mu_2)(u, v) & \text{if } (u, v) \in E_1 \cup E_2 \\ \\ \sigma_1(u) \wedge \sigma_2(v) & \text{if } (u, v) \in E^{'}. \end{cases}$$

Next we find the the strong domination polynomial of the join of two fuzzy graphs.

Theorem 3.8. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs with $|G_1^*| = m$ and $|G_2^*| = n$ with $V_1 \cap V_2 = \phi$ respectively. Then $D_s(G_1 + G_2, x) = ((1 + x)^m - 1)((1 + x)^n - 1) + D_s(G_1, x) + D_s(G_2, x)$.*

Proof. Suppose j is a natural number such that $1 \leq j \leq m + n$. We have to find $d(G_1 + G_2, j)$. If j is the sum of two natural numbers, say k and l,i.e., $j = k + l$ then clearly, for every $V_1 \subseteq V(G_1)$ and $V_2 \subseteq V(G_2)$ such that $|V_1| = k$, $|V_2| = l$, $V_1 \cup V_2$ is strong dominating set of $G_1 + G_2$. Also if $S \in D_s(G_1, j)$, then S is a strong dominating set for $G_1 + G_2$ of cardinality j. Similarly, for every $S \in D_s(G_2, j)$, $D_s(G_1 + G_2, x) = ((1 + x)^m - 1)((1 + x)^n - 1) + D_s(G_1, x) + D_s(G_2, x)$. □

The following corollary is an immediate consequence of Theorem 3.8. This corollary gives a formula for the strong domination polynomial of complete bipartite fuzzy graph, star fuzzy graph and the wheel fuzzy graph.

Corollary 3.9. (1) *In the complete bipartite fuzzy graph* K_{σ_1,σ_2},

$$D_s(K_{\sigma_1,\sigma_2}) = ((1+x)^{n_1} - 1)((1+x)^{n_2} - 1) + x^{n_1} + x^{n_2},$$

where n_1 *and* n_2 *are the cardinalities of the bipartition* V_1 *and* V_2.

(2) *In the star fuzzy graph,*

$$D_s(K_{\sigma_1,\sigma_2}) = x^{n_2} + x(1+x)^{n_2},$$

where $|V_1| = 1$ *and* $|V_2| = n_2$.

(2) *For* $m \geq 4$, *if* W_σ *is a wheel fuzzy graph with* m *nodes, then*

$$D_s(W_\sigma, x) = x(1+x)^{m-1} + D_s(C, x),$$

where C *is a fuzzy cycle on* $m-1$ *nodes.*

Proof. (1) By applying Theorem 3.8 with $G_1 = \bar{K_\sigma}$ with n_1 nodes and $G_2 = \bar{K_\sigma}$ with n_2 nodes, the desired result is obtained.

(2) It is enough to apply part (1) for $n_1 = 1$.

(3) Since for every $m \geq 4$, $W_\sigma = C + K_\sigma$, where W_σ has m nodes, C has $m-1$ nodes and K_σ has one node. Then the desired result is obtained by Theorem 3.8. □

In Corollary 2(iii) we derived a relationship between the strong domination polynomial of fuzzy wheels and fuzzy cycles.

4. Coefficients of strong domination polynomial

In this section, it is obtained some properties of the coefficients of the strong domination polynomial of a fuzzy graph. The next theorem is obtained as an obvious consequence from the definition of the strong domination polynomial of a fuzzy graph.

Theorem 4.1. *Let* $G : (V, \sigma, \mu)$ *is a fuzzy graph with* $|V(G)| = n$.

(1) *If* G *is connected, then* $d_s(G, n) = 1$ *and* $d_s(G, n-1) = n$.

(2) $d_s(G, i) = 0$ *if and only if* $i < \gamma_s(G)$ *or* $i > n$.

(3) $D_s(G, x)$ *has no constant term.*

(4) $D_s(G, x)$ *is a strictly increasing function in* $[0, \infty)$.

(5) *Let* $G : V, \sigma, \mu)$ *be a fuzzy graph and* H *be any induced fuzzy subgraph of* G. *Then* $deg(D_s(G, x)) \geq Deg(D_s(H, x))$.

(6) *Zero is a zero of* $D_s(G, x)$ *with multiplicity* $\gamma_s(G)$.

Strong domination polynomial of composition of two fuzzy graphs

Definition 4.2. Let $G : (V, \sigma, \mu)$ be any fuzzy graph with node set $v_1, v_2, ...v_n$. Add n new nodes $u_1, u_2, ...u_n$ and join u_i to v_i with $\mu(u_i, v_i) = \wedge(\sigma(u_i), \sigma(v_i))$, for $1 \leq i \leq n$. We shall represent this fuzzy graph by $G \centerdot K_1$, where K_1 is a complete fuzzy graph on one node.

Next, we will find the strong domination polynomial of $G \centerdot K_1$.

Theorem 4.3. *For any fuzzy graph* G *of* n *nodes,* $\gamma_s(G \centerdot K_1) = n$.

Proof. If S is a strong dominating set of $G \centerdot K_1$, then for every $1 \leq i \leq n$, $u_i \in S$ or $v_i \in S$. Thus $|S| \geq n$. Since by definition, $u_1, u_2, ...u_n$ is a strong dominating set of $G \centerdot K_1$, we have $\gamma_s(G \centerdot K_1) = n$. □

Remark 4.4. By Theorem 4.3, $d_s(G \bullet K_1, m) = 0$ for $m < n$, since $\gamma_s(G \bullet K_1) = n$. Then there does not exist any strong dominating set of cardinality m. Thus in the next theorem, it is computed $d_s(G \bullet K_1, m)$ for $n \leq m \leq 2n$.

Theorem 4.5. *For any fuzzy graph $G : (V, \sigma, \mu)$ with n number of nodes and $n \leq m \leq 2n$, $d_s(G \bullet K_1, m) = \binom{n}{m-n} 2^{2n-m}$.*

Proof. Let us assume that S is a strong dominating set of $G \bullet K_1$ with m nodes and $|V(G) \bigcap S| = i$ for $0 \leq i \leq n$. Without loss of generality, we can assume that $V(G) \bigcap S = \{v_1, v_2, ...v_i\}$. Then S contains $u_{i+1}, ...u_n$. For extending

$$\{v_1, v_2, ...v_i, u_{i+1}, u_{i+2}....u_n\}$$

to a strong dominating set S of cardinality m with $V(G) \bigcap S = \{v_1, v_2, ...v_i\}$, we have $\binom{i}{m-n}$ possibilities. Thus we have

$$d_s(G \bullet K_1, m) = \sum_{i=0}^{n} \binom{n}{i} \binom{i}{m-n} = \binom{n}{m-n} 2^{2n-m}.$$

□

Example 4.6. Consider a fuzzy graph $G : (V, \sigma, \mu)$ on 2 nodes which are joined by a strong arc. Then $G \bullet K_1$ has 4 nodes say v_1, v_2, u_1, u_2. Then its strong dominating sets of cardinality 2 are $\{v_1, v_2\}$, $\{u_1, u_2\}$, $\{v_1, u_2\}$, $\{v_2, u_1\}$. Thus $d_s(G, 2) = 4 = \binom{2}{0} 2^{2*2-2}$. The strong dominating sets of cardinality 3 are $\{v_1, v_2, u_1\}$, $\{v_2, u_1, u_2\}$, $\{v_1, v_2, u_2\}$, $\{v_1, u_1, u_2\}$. So $d_s(G, 3) = 4 = \binom{2}{1} 2^{2*2-3}$. The strong dominating sets of cardinality 4 is $\{v_1, v_2, u_1, u_2\}$. Hence $d_s(G, 4) = 1 = \binom{2}{2} 2^{2*2-4}$.

Theorem 4.7. *In any fuzzy graph $G : (V, \sigma, \mu)$ with n nodes, $d_s(G, n) = 1$. If G has no isolated node, then $d_s(G, n-1) = n$.*

Proof. Its an obvious result from the definition of $d_s(G, m)$. □

5. Conclusion

Fuzzy graph polynomials are a well-developed area useful for analyzing properties of fuzzy graphs. There are some polynomials associated to fuzzy graphs. Chromatic polynomial, clique polynomial, characteristic polynomial and Tutte polynomial are some examples of these polynomials. By the analysis of fuzzy graph polynomial and studying its properties we get some information about the fuzzy graph, which motivated us to introduce this new type of fuzzy graph polynomial. This paper introduce the domination polynomial of a fuzzy graph using strong arcs, investigate its properties and establish some relationships between this polynomial and domination number of fuzzy graph. The domination polynomial of classes of fuzzy graphs is obtained. Also it is obtained the strong domination polynomial of composition of two fuzzy graphs.

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