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Abstract. This paper analyses the fuzzy degree of a particular genetic hypergroup $(H, \otimes)$, associated to non-Mendelian inherence. It is considered a sequence of membership functions and of join spaces, obtained starting with a hypergroupoid $(H, \otimes)$ (See [1]). The fuzzy grade is the minumum natural number i such that two consecutive associated join spaces, of the above mentioned sequence, H_i and H_{i+1} are isomorphic.

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1. Introduction

Theory of hyperstructures is a field of algebra, more than 70 years old and very rich in applications, for instance in geometry, fuzzy and rough sets, automata, cryptography, codes, probabilities, graphs and hypergraphs (See [2]). Connections between hypergroups and genetics were analyzed in [3, 4]. We analyze here one of these connections, namely between hypergroup theory with non-Mendelian inherence.

2. Preliminaries

First, recall some basic definitions in hypergroups, that we shall use in what follows:

A *hyperoperation* on a nonempty set H is a map $\circ : H \times H \to P^*(H)$, where $P^*(H)$ denotes the set of nonempty subsets of H.

For subsets $A,\ B$ of H, set $A \circ B = \bigcup_{a \in A; b \in B} a \circ b$, and for $h \in H$ write $h \circ A$ and $A \circ h$ for $\{h\} \circ A$ and $A \circ \{h\}$.

The pair $(H, \circ)$ is a *hypergroup*, if for all a, b, c of H we have

$$(a \circ b) \circ c = a \circ (b \circ c) \text{ and } a \circ H = H \circ a = H.$$

If only the associativity is satisfied, then $(H, \circ)$ is a *semihypergroup*. The condition $a \circ H = H \circ a = H$ for all a of H is called the *reproductive law.*

A nonempty subset K of H is a *subhypergroup*, if $K \circ K \subseteq K$ and for all $a \in K$, $K \circ a = K = a \circ K$.

A commutative hypergroup $(H, \circ)$ is a *join space*, if the following implication holds: for all a, b, c, d, x of H,

$$a \in b \circ x, c \in d \circ x \Rightarrow a \circ d \cap b \circ c \neq \varnothing.$$

A *semijoin space* is a commutative semihypergroup satisfying the join condition.

Join spaces are an important tool in the study of graphs and hypergraphs, binary relations, fuzzy and rough sets and in the reconstruction of several types of non-euclidean geometries, such as the descriptive, spherical and projective geometries. Several interesting books have been written on hyperstructures and their applications (See for instance [2, 5, 6, 7]).

Let us recall now the fuzzy degree notion. In 2003, Corsini proved that with every hypergroupoid $(H, \otimes)$ one can associate a fuzzy subset, as follows (See [1]):

$$\forall u \in H,\ Q(u) = \{(x, y) \in H^2 \mid u \in x \otimes y\},$$
$$q(u) = \mid Q(u) \mid,$$
$$\alpha(u) = \sum_{(x,y)\in Q(u)} 1/ \mid x \otimes y \mid,$$
$$\mu(u) = \alpha(u)/q(u). \qquad (*)$$

Now we can associate a new join space structure $(H, \circ_\mu)$ denoted also by $(H, \circ)$, which is associated to the fuzzy set μ as follows:

$$x \circ y = \{z \mid min\{\mu(x), \mu(y)\} \leq \mu(z) \leq max\{\mu(x), \mu(y)\} \qquad (**)$$

In this manner we obtain a sequence of join spaces and fuzzy sets and the fuzzy degree of the starting hypergroup is the minimum natural number for which the hypergroups associated to fuzzy sets as in $(*)$ are isomorphic.

This problem has been studied using fuzzy sets endowed with two membership functions by Cristea [8], in connection with reduced hypergroups by Stefanescu-Cristea [9] and for various classes of hypergroups: Corsini-Cristea for i.p.s. hypergroups (a particular case of Canonical hypergroups) [10], and 1-hypergroups (hypergroups for which the heart is a singleton) [11], Corsini, Leoreanu, Iranmanesh for hypergroups associated with hypergraphs (See [12]). Atanassov's intuitionistic fuzzy grade of hypergroups was analized in [13].

In this paper we consider the above construction for a particular genetic hypergroup, in order to obtain its fuzzy degree in several particular cases, that occour in genetics.

3. Connections between hypergroups and genetics

In [3], Tahan and Davvaz analysed some examples of five different types of non-Mendelian inheritance : Epistasis, Supplementary gene, Inhibitory gene, Complementary gene, Supplementary and complementary gene) and connect them to hypergroup theory. We analyse here the first type Epistasis, which provides us the most interesting hypergroup structure among the above mentioned types.

Consider the epistasis of dominant gene in the coat color of dogs. There are two allelomorphic pairs Aa and Bb, where A and B are dominant over a and b respectively. Summarizing this experiment, we obtain:

$$P: \ AABB \otimes aabb$$
$$F_1: \ AaBb$$

and

$$F_1 \otimes F_1: \ AaBb \otimes AaBb$$
$$F_2: White, Black, Brown.$$

A_1 denotes White, A_2 denotes Black and A_3 denotes Brown.

Set $H = \{A_1, A_2, A_3\}$. We obtain the following hypergroup:

$\otimes$	A_1	A_2	A_3
A_1	H	H	H
A_2	H	$\{A_2, A_3\}$	$\{A_2, A_3\}$
A_3	H	$\{A_2, A_3\}$	$\{A_3\}$

Table 1

In what follows, we generalize the above hypergroup to a hypergoup of n elements, where $n \geq 3$ in the next manner: $H = \{A_1, A_2, ..., A_n\}$ and

$\otimes$	A_1	A_2	A_3	$\cdots$	A_n
A_1	H	H	H	$\cdots$	H
A_2	H	$\{A_2, A_3, \cdots, A_n\}$	$\{A_2, A_3, \cdots, A_n\}$	$\cdots$	$\{A_2, A_3, \cdots, A_n\}$
A_3	H	$\{A_2, A_3, \cdots, A_n\}$	$\{A_3, \cdots, A_n\}$	$\cdots$	$\{A_3, \cdots, A_n\}$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
A_n	H	$\{A_2, A_3, \cdots, A_n\}$	$\{A_3, \cdots, A_n\}$	$\cdots$	$\{A_n\}$

Table 2

Let us present some properties of this hypergroup.

Theorem 3.1. (1) *The hyperoperation $\otimes$ is commutative.*
(2) *For all $1 \leq i \leq j \leq k \leq n$ we have $A_i \otimes (A_j \otimes A_k) = \{A_i, ..., A_n\}$.*
(3) *$(H, \otimes)$ is a join space.*
(4) *$(H, \otimes)$ is regular reversible, but it is not a feeble canonical hypergroup.*
(5) *The only complete part of $(H, \otimes)$ is H.*
(6) *$(H, \otimes)$ it is not complete.*
(7) *The subhypergroups of $(H, \otimes)$ have the form $S_k = \{A_i \mid i \geq k\}$, where $1 \leq k \leq n$. All these subhypergroups are not closed, except for $S_1 = H$.*

Proof. (3) The element A_n belongs to any hyperproduct, so the join space condition is satisfied.

(4) All elements of H are identities. Since for all $1 \leq i, \ j \leq n$, we have $A_i \in A_j \otimes A_i$. Moreover, the set of all inverses of an element is H. The hypergroup is not feebly canonical since $A_1, \ A_2$ are inverses of an arbitrary element A_k, but $A_1 \otimes A_n \neq A_2 \otimes A_n$.

(6) The hypergroup is not complete since for instance $A_2 \otimes A_3 \neq H$, which is the complete closure of $A_2 \otimes A_3$, because the heart of H is H itself. □

Applying Corsini's construction (*) for the studied hypergroup we obtain the following calculations.

4. Step 1

$Q(A_1) = \{(A_1, A), (A, A_1) \mid A \in H\}$ whence $q_1(A_1) = |Q(A_1)| = 2n - 1$.

$Q(A_2) = Q(A_1) \cup \{(A_2, A), (A, A_2) \mid A \in H\}$ whence $q_1(A_2) = |Q(A_2)| = 2n - 1 + 2n - 3 = 4n - 4$.

$Q(A_3) = Q(A_2) \cup \{(A_3, A), (A, A_3) \mid A \in H\}$ whence $q_1(A_3) = |Q(A_3)| = 2n - 1 + 2n - 3 + 2n - 5 = 6n - 9$.

$Q(A_k) = Q(A_{k-1}) \cup \{(A_k, A), (A, A_k) \mid A \in H\}$ whence $q_1(A_k) = |Q(A_k)| = 2n - 1 + 2n - 3 + 2n - 5 + ... + 2n - (2k - 1) = 2nk - k^2$.
Then for all natural number $k,\ 1 \leq k \leq n$,

$$q_1(A_k) = 2nk - k^2. \tag{1}$$

Finally, $q_1(A_n) = n^2$. For any $A \in H$, we have

$$\mu_1(A) = \alpha_1(A)/q_1(A). \tag{2}$$

Moreover, we have $\alpha_1(A_1) = (2n - 1)/n$, whence $\mu_1(A_1) = 1/n$. Also, we have

$$\alpha_1(A_2) = (2n - 1)/n + (2n - 3)/(n - 1) = (4n^2 - 6n + 1)/n(n - 1).$$

Thus $\mu_1(A_2) = (4n^2 - 6n + 1)/4n(n - 1)^2 > \mu_1(A_1) = 1/n$. So $\mu_1(A_2) > \mu_1(A_1)$.
Generally,

$$\alpha_1(A_k) = \alpha_1(A_{k-1}) + (2(n - (k - 1)) - 1)/(n - (k - 1)) \tag{3}$$

whence, by adding all the above equalities (2) for all $1 \leq j \leq k$, we obtain

$$\begin{aligned} \alpha(A_k) &= \alpha_1(A_1) + (2n - 3)/(n - 1) + (2n - 5)/(n - 2) + \cdots \\ &\quad + (2(n - (k - 1)) - 1)/(n - (k - 1)) \\ &= \textstyle\sum_{j=0}^{k-1} (2(n - j) - 1)/(n - j) \\ &= 2k - \textstyle\sum_{j=0}^{k-1} 1/(n - j). \text{ Denote } \sum_{j=1}^{n} 1/j = H_n. \end{aligned}$$

For $k = n$, we obtain $\alpha_1(A_n) = 2n - \sum_{j=0}^{n-1} 1/(n - j) = 2n - H_n$, where H_n is roughly estimated to $\gamma + \log n$ and $\gamma \approx 0.57721$ is the Euler-Mascheroni constant. Hence

$$\mu_1(A_n) = (2n - H_n)/n^2. \tag{4}$$

For $1 \leq k < n$, we have $\alpha_1(A_k) = 2k - (H_n - H_{n-k})$ which is roughly estimated to

$$2k - (log\ n + \gamma - log\ (n - k) - \gamma) = 2k - log\ n/(n - k).$$

Hence $\alpha_1(A_k) \approx 2k - log\ n/(n - k)$. Therefore for all $1 \leq k < n$,

$$\mu_1(A_k) = (2k - H_n + H_{n-k})/[k(2n - k)]. \tag{5}$$

Theorem 4.1. *For $1 \leq i < j \leq n$, we have $\mu_1(A_i) < \mu_1(A_j)$.*

Proof. Indeed, for $1 < k < n-1$, we have the following equivalences:

$\mu_1(A_k) < \mu_1(A_{k+1})$
$\Leftrightarrow (2k - H_n + H_{n-k})/[k(2n-k)] < (2k+2-H_n+H_{n-k-1})/[(k+1)(2n-k-1)]$
$\Leftrightarrow 2/(2n-k) - T_n/k(2n-k)$
$< 2/(2n-k-1) - [T_n + 1/(n-k)]/(k+1)(2n-k-1)$, where $T_n = H_n - H_{n-k}$.

Indeed, $H_n - H_{n-k-1} = T_n + 1/(n-k)$. Then we get

$\mu_1(A_k) < \mu_1(A_{k+1})$
$\Leftrightarrow 2/(2n-k) - 2/(2n-k-1) < T_n[1/k(2n-k) - 1/(k+1)(2n-k-1)]$
$-1/[(n-k)(k+1)(2n-k-1)]$
$\Leftrightarrow -2/(2n-k)(2n-k-1) < T_n[1/k(2n-k) - 1/(k+1)(2n-k-1)]$
$-1/[(n-k)(k+1)(2n-k-1)]$
$\Leftrightarrow 1/[(n-k)(k+1)(2n-k-1)] - 2/(2n-k)(2n-k-1)$
$< T_n[1/k(2n-k) - 1/(k+1)(2n-k-1)]$
$\Leftrightarrow k(2n-2k-1)/[(n-k)(k+1)(2n-k-1)(2n-k)]$
$> T_n[1/(k+1)(2n-k-1) - 1/k(2n-k)]$, which is true since
the lefthand term is positive, while the righthand term is negative.

Let us check now that $\mu(A_{n-1}) < \mu(A_n)$.

Indeed, $(2(n-1) - H_n + H_{n-(n-1)})/[(n-1)(2n-(n-1))] < (2n-H_n)/n^2$
$\Leftrightarrow (2n-1-H_n)/(n^2-1) < (2n-H_n)/n^2$
$\Leftrightarrow (-n+2)/[n(n^2-1)] < H_n/[n^2(n^2-1)]$, which is true for $n \geq 3$.

Thus we obtain the conclusion. □

Now, taking account of (**) we obtain a new hypergroup $(H, \circ_1)$ in which for all $A_i \in H$ we have

$A_i \circ_1 A_i = \{a_i\}$ and for all $1 \leq i < j \leq n$ we have $A_i \circ_1 A_j = A_j \circ_1 A_i = \{A_i, A_{i+1}, ..., A_j\}$. Then

$\circ_1$	A_1	A_2	A_3	$\cdots$	A_n
A_1	A_1	$\{A_1, A_2\}$	$\{A_1, A_2, A_3\}$	$\cdots$	H
A_2		A_2	$\{A_2, A_3\}$	$\cdots$	$\{A_2, A_3, \cdots, A_n\}$
A_3			A_3	$\cdots$	$\{A_3, \cdots, A_n\}$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
A_n				$\cdots$	A_n

Table 3

5. Step 2

Applying (*), we obtain the following:

$q_2(A_1) = 2n-1,\ q_2(A_2) = 2 \cdot 2 \cdot (n-1) - 1 = 4n-5,$
$q_2(A_3) = 2 \cdot 3 \cdot (n-2) - 1 = 6n - 13,\ q_2(A_4) = 2 \cdot 4 \cdot (n-3) - 1 = 8n - 25$ and for $1 \leq k \leq n$,

$$q_2(A_k) = 2 \cdot k \cdot (n-k+1) - 1. \tag{6}$$

Particularly, $q_2(A_n) = 2n - 1 = q(A_1),\ q_2(A_{n-1}) = q_2(A_2) = 4(n-1) - 1,$

$q_2(A_{n-2}) = q_2(A_3) = 2 \cdot 3 \cdot (n-2) - 1.$

If $n = 2s$, where s is a natural number, $s \geq 2$ then

$$q_2(A_s) = q_2(A_{s+1}) = 2s(s+1) - 1. \tag{7}$$

If $n = 2s+1$, where s is a natural number, $s \geq 1$ then

$$q_2(A_s) = q_2(A_{s+2}) = 2s(s+2) - 1 \tag{8}$$

and

$$q_2(A_{s+1}) = 2(s+1)^2 - 1. \tag{9}$$

For any $A \in H$ we have

$$\mu_2(A) = \alpha_2(A)/q_2(A). \tag{10}$$

Let us calculate now $\alpha_2(A_k)$ for $1 \leq k \leq n$.

Case 1

For $n = 2s$ we have:

$\alpha_2(A_1) = \alpha_2(A_{2s}) = 1 + 2/2 + 2/3 + \cdots + 2/n = 1 + 2/2 + 2/3 + \cdots + 2/(2s)$
$\alpha_2(A_2) = \alpha_2(A_{2s-1}) = 1 + 4/2 + 4/3 + ... + 4/(2s-1) + 2/(2s)$
$\alpha_2(A_3) = \alpha_2(A_{2s-2}) = 1 + 4/2 + 6/3 + 6/4 + \cdots + 6/(2s-2) + 4/(2s-1) + 2/(2s)$

$\cdots\cdots\cdots$

$\alpha_2(A_s) = \alpha_2(A_{s+1}) = 1 + 4/2 + 6/3 + \cdots + (2s-2)/(s-1) + 2s/s$
$+2s/(s+1) + (2s-2)/(s+2) + \cdots + 2/(2s).$

It follows that

$$\mu_2(A_1) = \mu_2(A_{2s}),\ \mu_2(A_2) = \mu_2(A_{2s-1}), \cdots, \mu_2(A_s) = \mu_2(A_{s+1}). \tag{11}$$

By calculations, we get that $\mu_2(A_1) > \mu_2(A_2) > \cdots > \mu_2(A_s)$.

Case 2

For $n = 2s+1$ we have:
$\alpha_2(A_1) = \alpha_2(A_{2s+1}) = 1 + 2/2 + 2/3 + \cdots + 2/(2s+1)$
$\alpha_2(A_2) = \alpha_2(A_{2s}) = 1 + 4/2 + 4/3 + \cdots + 4/(2s-1) + 4/(2s) + 2/(2s+1)$
$\alpha_2(A_3) = \alpha_2(A_{2s-1}) = 1 + 4/2 + 6/3 + 6/4 + \cdots + 6/(2s-1) + 4/(2s) + 2/(2s+1)$

$\cdots\cdots\cdots$

$\alpha_2(A_s) = \alpha_2(A_{s+2}) = 1 + 4/2 + 6/3 + \cdots + (2s-2)/(s-1) + 2s/s$
$+2s/(s+1) + 2s/(s+2) + (2s-2)/(s+3) + \cdots + 4/2s + 2/(2s+1)$
$\alpha_2(A_{s+1}) = 1 + 4/2 + 6/3 + \cdots + 2s/s + (2s+2)/(s+1) + 2s/(s+2) +$
$\cdots + 4/2s + 2/(2s+1).$

It follows that

$$\mu_2(A_1) = \mu_2(A_{2s+1}),\ \mu_2(A_2) = \mu_2(A_{2s}), \cdots, \mu_2(A_s) = \mu_2(A_{s+2}). \tag{12}$$

By calculations, we get that $\mu_2(A_1) > \mu_2(A_2) > \cdots > \mu_2(A_s) > \mu_2(A_{s+1})$.

We can conclude this step as follows:

Theorem 5.1.

(1) *For $n = 2s$ we have:*

$$\mu_2(A_1) = \mu_2(A_{2s}),\ \mu_2(A_2) = \mu_2(A_{2s-1}), \cdots, \mu_2(A_s) = \mu_2(A_{s+1})$$

and $\mu_2(A_1) > \mu_2(A_2) > \cdots > \mu_2(A_s)$.

(2) *For $n = 2s+1$ we have:*

$$\mu_2(A_1) = \mu_2(A_{2s+1}),\ \mu_2(A_2) = \mu_2(A_{2s}), \cdots, \mu_2(A_s) = \mu_2(A_{s+2})$$

and $\mu_2(A_1) > \mu_2(A_2) > \cdots > \mu_2(A_s) > \mu_2(A_{s+1})$.

Now, taking account of (**), we obtain a new hypergroup $(H, \circ_2)$ in which for all $A_i \in H$ we have $A_i \circ_2 A_i = \{A_i, A_{n+1-i}\}$. Moreover, for all $1 \leq i < j \leq n$ we have $A_i \circ_2 A_j = A_j \circ_2 A_i = \{A_i, A_{n+1-i}, A_{i+1}, A_{n-i}, \cdots, A_j, A_{n+1-j}\}$.

Denote $\hat{A}_i = \{A_i, A_{n+1-i}\}$. Hence

$$A_i \circ_2 A_i = \hat{A}_i, \quad A_i \circ_2 A_j = A_j \circ_2 A_i = \hat{A}_i \cup \hat{A_{i+1}} \cup \cup \hat{A}_j.$$

In what follows we analyze separately the cases n even and n odd.

6. Step 3

Case $n = 2s$.

$\circ_2$	A_1	A_2	$\cdots$	A_s	A_{s+1}	$\cdots$	A_{2s-1}	A_{2s}
A_1	$\hat{A}_1$	$\hat{A}_1 \cup \hat{A}_2$	$\cdots$	$\hat{A}_1 \cup \cdots \cup \hat{A}_s$	$\hat{A}_1 \cup \cdots \cup \hat{A}_s$	$\cdots$	$\hat{A}_1 \cup \hat{A}_2$	$\hat{A}_1$
A_2		$\hat{A}_2$	$\cdots$	$\hat{A}_2 \cup \cdots \cup \hat{A}_s$	$\hat{A}_2 \cup \cdots \cup \hat{A}_s$	$\cdots$	$\hat{A}_2$	$\hat{A}_1 \cup \hat{A}_2$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
A_s				$\hat{A}_s$	$\hat{A}_s$	$\cdots$	$\cdots$	$\hat{A}_1 \cup \cdots \cup \hat{A}_s$
A_{s+1}				$\hat{A}_s$	$\hat{A}_s$	$\cdots$	$\cdots$	$\hat{A}_1 \cup \cdots \cup \hat{A}_s$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
A_{2s}	$\hat{A}_1$	$\hat{A}_1 \cup \hat{A}_2$		$\hat{A}_1 \cup \cdots \cup \hat{A}_s$	$\hat{A}_1 \cup \cdots \cup \hat{A}_s$		$\hat{A}_1 \cup \hat{A}_2$	$\hat{A}_1$

Table 4

Notice a double symmetry of the table horizontally and vertically. Then we have:
$q_3(A_1) = 4(2s-1) = 8s-4 = q_3(A_{2s})$,
$q_3(A_2) = 4[2 \cdot 2(s-1) - 1] = 4(4s-5) = q_3(A_{2s-1})$.
$q_3(A_k) = 4[2 \cdot k \cdot (s-k+1) - 1] = q_3(A_{2s-k+1})$, for $1 \leq k \leq s$.

Particularly, $q_3(A_s) = 4(2s-1) = q_3(A_{s+1}) = q_3(A_1)$. Similarly, $q_3(A_{s-1}) = q_3(A_2)$ and so on, similarly as in Step 2.

According to (*),

$$\forall A \in H,\ \mu_3(A) = \alpha_3(A)/q_3(A). \tag{13}$$

Let us calculate now $\alpha_3(A)$.

We have $\alpha_3(A_1) = \alpha_3(A_s) = 4[1/2 + 2/4 + 2/6 + ... + 2/(2s-1) + 2/(2s)]$,

$\alpha_3(A_2) = \alpha_3(A_{s-1}) = 4[1/2 + 4/4 + ... + 4/(2s-2) + 2/(2s)]$ and so on, similarly as in Step 2. Notice that $|A_i \circ_2 A_j| = 2|A_i \circ_1 A_j|$, for all indices i, j. Also, we can distinguish between cases when s is even or odd as in Step 2.

We finally obtain the followings:

If s is even, then $\mu_3(A_1) = \mu_3(A_s),\ \mu_3(A_2) = \mu_3(A_{s-1}), \cdots, \mu_3(A_{s/2}) = \mu_3(A_{1+s/2})$.

If s is odd, then $\mu_3(A_1) = \mu_3(A_s),\ \mu_3(A_2) = \mu_3(A_{s-1}), \cdots, \mu_3(A_{(s-1)/2}) = \mu_3(A_{(s+3)/2})$.

Case $n = 2s + 1$.

Clearly, if $n = 2s + 1$, then $\hat{A_{s+1}} = \{A_{s+1}\}$. Notice that this is the only class containing only one element.

$\circ_2$	A_1	A_2	$\cdots$	A_s	A_{s+1}	A_{s+2}	$\cdots$	A_{2s+1}
A_1	$\hat{A_1}$	$\hat{A_1} \cup \hat{A_2}$	$\cdots$	$\hat{A_1} \cup \cdots \cup \hat{A_s}$	$\hat{A_1} \cup \cdots \cup \hat{A_{s+1}}$	$\hat{A_1} \cup \cdots \cup \hat{A_s}$	$\cdots$	$\hat{A_1}$
A_2		$\hat{A_2}$	$\cdots$	$\hat{A_2} \cup \cdots \cup \hat{A_s}$	$\hat{A_2} \cup \cdots \cup \hat{A_{s+1}}$	$\hat{A_2} \cup \cdots \cup \hat{A_s}$	$\cdots$	$\hat{A_1} \cup \hat{A_2}$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
A_s				$\hat{A_s}$	$\hat{A_s} \cup \hat{A_{s+1}}$	$\hat{A_s}$	$\cdots$	$\hat{A_1} \cup \cdots \cup \hat{A_s}$
A_{s+1}					$\hat{A_{s+1}}$	$\hat{A_s} \cup \hat{A_{s+1}}$	$\cdots$	$\hat{A_1} \cup \cdots \cup \hat{A_s}$
...	...	...	...	...	...	...	...	...
A_{2s+1}								$\hat{A_1}$

Table 5

The column and the line corresponding to element A_{s+1} represent the axes of symmetry of the table. Thus we get

$q_3(A_1) = 4(2s-1) + 4 = 8s = q_3(A_{2s+1})$,

$q_3(A_2) = 16s - 12 = q_3(A_{2s})$,

$q_3(A_k) = 4[2 \cdot k \cdot (s-k+1) - 1] + 4 = q_3(A_{2s-k+2})$ for $1 \leq k \leq s$.

Particularly, $q_3(A_s) = 12s - 4 = q_3(A_{s+2})$. Similarly, $q_3(A_{s-1}) = q_3(A_{s+3})$ and so on, similarly as in Step 2.

Finally, $q_3(A_{s+1}) = 4s + 1$.

Let us calculate now $\alpha_3(A)$ for $A \in H$.

We have: $\alpha_3(A_s) = \alpha_3(A_{s+2}) = [4/2 + 8/4 + ...8/(2s)] + 4/(2s-u)... + 4/(2s-1) + 4/(2s+1)$, where u is odd and $|\{2s-u, ..., 2s-1, 2s+1\}| = s$. Finally,

$\alpha_3(A_1) = \alpha_3(A_{2s+1}) = 4[1/2 + 2/4 + ... + 2/2s] + 4/(2s+1)$ and so on, similarly as in Step 2.

Moreover, $\alpha_3(A_{s+1}) = 1 + 4/3 + 4/5 + ... + 4/(2s+1)$. Notice that the element A_{s+1} appears only in column and in line $s + 1$.

Now, $\forall A \in H,\ \mu_3(A) = \alpha_3(A)/q_3(A)$. We have

$\mu_3(A_{s+1}) = \alpha_3(A_{s+1})/q_3(A_{s+1}) = [1 + 4/3 + 4/5 + ... + 4/(2s+1)]/(4s+1)$.

By calculations, we finally obtain that

$$\mu_3(A_{s+1}) > \mu_3(A_s) > \cdots > \mu_3(A_2) > \mu_3(A_1).$$

We check that for $n \in \{5, 7\}$ the fuzzy degree of $(H, \otimes)$ is 3.

7. Examples

Example 7.1. Let $n = 6$. Then $s = 3$. We have the following table for $\circ_1$:

$\circ_1$	A_1	A_2	A_3	$\cdots$	A_6
A_1	A_1	$\{A_1, A_2\}$	$\{A_1, A_2, A_3\}$	$\cdots$	H
A_2		A_2	$\{A_2, A_3\}$	$\cdots$	$\{A_2, A_3, \cdots, A_6\}$
A_3			A_3	$\cdots$	$\{A_3, \cdots, A_6\}$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
A_6				$\cdots$	A_6

Table 6

By calculation, we have

$$\begin{aligned}\mu_2(A_1) = \mu_2(A_6) &= 3.9/11 = 0.354\\ &> \mu_2(A_2) = \mu_2(A_5) = 6.46/19 = 0.34\\ &> \mu_2(A_3) = \mu_2(A_4) = 7.63/25 = 0.305.\end{aligned}$$

Then we obtain the following table:

$\circ_2$	A_1	A_2	A_3	A_4	A_5	A_6
A_1	$\hat{A}_1$	$\hat{A}_1 \cup \hat{A}_2$	$\hat{A}_1 \cup \hat{A}_2 \cup \hat{A}_3$	$\hat{A}_1 \cup \hat{A}_2 \cup \hat{A}_3$	$\hat{A}_1 \cup \hat{A}_2$	$\hat{A}_1$
A_2		$\hat{A}_2$	$\hat{A}_2 \cup \hat{A}_3$	$\hat{A}_2 \cup \hat{A}_3$	$\hat{A}_2$	$\hat{A}_1 \cup \hat{A}_2$
A_3			$\hat{A}_3$	$\hat{A}_3$	$\hat{A}_2 \cup \hat{A}_3$	$\hat{A}_1 \cup \hat{A}_2 \cup \hat{A}_3$
A_4				$\hat{A}_3$	$\hat{A}_2 \cup \hat{A}_3$	$\hat{A}_1 \cup \hat{A}_2 \cup \hat{A}_3$
A_5					$\hat{A}_2$	$\hat{A}_1 \cup \hat{A}_2$
A_6						$\hat{A}_1$

Table 7

By calculation, we get

$$\begin{aligned}\mu_3(A_1) = \mu_3(A_6) = \mu_3(A_3) = \mu_3(A_4) &= 5,32/20 = 0.266\\ &> \mu_3(A_2) = \mu_3(A_5) = 7.32/28 = 0.261.\end{aligned}$$

Denote $[A_1] = \{A_1, A_3, A_4, A_6\}$, $[A_2] = \{A_2, A_5\}$. Thus we obtain the following table:

$\circ_3$	A_1	A_2	A_3	A_4	A_5	A_6
A_1	$[A_1]$	H	$[A_1]$	$[A_1]$	H	$[A_1]$
A_2	H	$[A_2]$	H	H	$[A_2]$	H
A_3	$[A_1]$	H	$[A_1]$	$[A_1]$	H	$[A_1]$
A_4	$[A_1]$	H	$[A_1]$	$[A_1]$	H	$[A_1]$
A_5	H	$[A_2]$	H	H	$[A_2]$	H
A_6	$[A_1]$	H	$[A_1]$	$[A_1]$	H	$[A_1]$

Table 8

By calculation, we have

$$\mu_4(A_1) = \mu_4(A_6) = \mu_4(A_3) = \mu_4(A_4) = 6.66/32 = 0.208$$

$< \mu_4(A_2) = \mu_4(A_5) = 4,66/20 = 0.233.$

So $(H, \circ_3)$ is isomorphic to $(H, \circ_4)$ and hence the fuzzy grade is 3.

Example 7.2. Let $n = 8$. Then $s = 4$.

$\otimes$	A_1	A_2	$\cdots$	A_6	A_7	A_8
A_1	H	H	$\cdots$	H	H	H
A_2	H	$A_2, \cdots, A_8$	$\cdots$	$A_2, \cdots, A_8$	$A_2, \cdots, A_8$	$A_2, \cdots, A_8$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
A_6	H	$A_2, \cdots, A_8$	$\cdots$	A_6, A_7, A_8	A_6, A_7, A_8	A_6, A_7, A_8
A_7	H	$A_2, \cdots, A_8$		A_6, A_7, A_8	A_7, A_8	A_7, A_8
A_8	H	$A_2, \cdots, A_8$	$\cdots$	A_6, A_7, A_8	A_7, A_8	A_8

Table 9

By calculation, we have

$$\begin{aligned}
\mu_1(A_1) = 1/8 = 0.125 &< \mu_1(A_2) = 0.1389 \\
&< \mu_1(A_3) = 5.562/39 = 0.142 \\
&< \mu_1(A_4) = 7.362/48 = 0.153 \\
&< \mu_1(A_5) = 9.112/55 = 0.165 \\
&< \mu_1(A_6) = 10.778/60 = 0.179 \\
&< \mu_1(A_7) = 12.278/63 = 0.194 \\
&< \mu_1(A_8) = 13.278/64 = 0.207.
\end{aligned}$$

Thus we obtain the next hypergroup structure, according to Step 1:

$\circ_1$	A_1	A_2	A_3	$\cdots$	A_8
A_1	A_1	$\{A_1, A_2\}$	$\{A_1, A_2, A_3\}$	$\cdots$	H
A_2		A_2	$\{A_2, A_3\}$	$\cdots$	$\{A_2, A_3, \cdots, A_8\}$
A_3			A_3	$\cdots$	$\{A_3, \cdots, A_8\}$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
A_8				$\cdots$	A_8

Table 10

Also, by calculation, we have

$$\begin{aligned}
\mu_2(A_1) = \mu_2(A_8) = 4.425/15 = 0.295 & \\
&> \mu_1(A_2) = \mu_2(A_7) = 7.61/27 = 0.281 \\
&> \mu_1(A_3) = \mu_2(A_6) = 9.52/35 = 0.272 \\
&> \mu_1(A_4) = \mu_2(A_5) = 10.42/39 = 0.267.
\end{aligned}$$

So we obtain the following table:

$\circ_2$	A_1	A_2	$\cdots$	A_4	A_5	$\cdots$	A_7	A_8
A_1	$\hat{A}_1$	$\hat{A}_1 \cup \hat{A}_2$	$\cdots$	$\hat{A}_1 \cup \cdots \cup \hat{A}_s$	$\hat{A}_1 \cup \cdots \cup \hat{A}_4$	$\cdots$	$\hat{A}_1 \cup \hat{A}_2$	$\hat{A}_1$
A_2		$\hat{A}_2$	$\cdots$	$\hat{A}_2 \cup \cdots \cup \hat{A}_4$	$\hat{A}_2 \cup \cdots \cup \hat{A}_4$	$\cdots$	$\hat{A}_2$	$\hat{A}_1 \cup \hat{A}_2$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
A_4				$\hat{A}_4$	$\hat{A}_4$	$\cdots$	$\cdots$	$\hat{A}_1 \cup \cdots \cup \hat{A}_4$
A_5				$\hat{A}_4$	$\hat{A}_4$	$\cdots$	$\cdots$	$\hat{A}_1 \cup \cdots \cup \hat{A}_4$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
A_8	$\hat{A}_1$	$\hat{A}_1 \cup \hat{A}_2$		$\hat{A}_1 \cup \cdots \cup \hat{A}_4$	$\hat{A}_1 \cup \cdots \cup \hat{A}_4$		$\hat{A}_1 \cup \hat{A}_2$	$\hat{A}_1$

Table 11

On the other hand, we have

$$\mu_3(A_1) = \mu_3(A_8) = \mu_3(A_4) = \mu_3(A_5) = 6.32/28 = 0.225$$
$$> \mu_3(A_2) = \mu_3(A_7) = \mu_3(A_3) = \mu_3(A_6) = 9.64/44 = 0.219.$$

Denote $[A_1] = \{A_1, A_4, A_5, A_8\}$ and $[A_2] = \{A_2, A_3, A_6, A_7\}$. This means that

$\circ_3$	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
A_1	$[A_1]$	H	H	$[A_1]$	$[A_1]$	H	H	$[A_1]$
A_2	H	$[A_2]$	$[A_2]$	H	H	$[A_2]$	$[A_2]$	H
A_3	H	$[A_2]$	$[A_2]$	H	H	$[A_2]$	$[A_2]$	H
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
A_8	$[A_1]$	H	H	$[A_1]$	$[A_1]$	H	H	$[A_1]$

Table 12

At the next step, we obtain the total hypergroup, acording to (**) that is $A_i \circ_4 A_j = H$ for all $i,\ j \in \{1, \cdots, 8\}$. Hence the fuzzy degree of this hypergroup is 4.

Example 7.3. Let $n = 5$. Then $s = 2$. We mention here only the table and calculations from step 2 on.

$\circ_2$	A_1	A_2	A_3	A_4	A_5
A_1	$\hat{A_1}$	$H - \{A_3\}$	H	$H - \{A_3\}$	$\hat{A_1}$
A_2	$H - \{A_3\}$	$\hat{A_2}$	$H - \hat{A_1}$	$\hat{A_2}$	$H - \{A_3\}$
A_3	H	$H - \hat{A_1}$	$\{A_3\}$	$H - \hat{A_1}$	H
A_4	$H - \{A_3\}$	$\hat{A_2}$	$H - \hat{A_1}$	$\hat{A_2}$	$H - \{A_3\}$
A_5	$\hat{A_1}$	$H - \{A_3\}$	H	$H - \{A_3\}$	$\hat{A_1}$

Table 13

The we get

$$\mu_3(A_3) = 3.13/9 = 0.347$$
$$> \mu_3(A_2) = \mu_3(A_4) = 6.13/20 = 0.306$$
$$> \mu_3(A_1) = \mu_3(A_5) = 4.8/16 = 0.3,$$

which holds also for index 4, i.e., $\mu_4(A_3) > \mu_4(A_2) = \mu_4(A_4) > \mu_4(A_1) = \mu_4(A_5)$. Thus the fuzzy degree $(H, \otimes)$ is 3.

Example 7.4. Let $n = 7$. Then $s = 3$.

$\otimes$	A_1	A_2	$\cdots$	A_6	A_7
A_1	H	H	$\cdots$	H	H
A_2	H	$A_2, \cdots, A_7$	$\cdots$	$A_2, \cdots, A_7$	$A_2, \cdots, A_7$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
A_6	H	$A_2, \cdots, A_7$	$\cdots$	A_6, A_7	A_6, A_7
A_7	H	$A_2, \cdots, A_7$		A_6, A_7	A_7

Table 14

Then we have

$$\mu_1(A_1) = 1/7 = 0.142$$
$$< \mu_1(A_2) = 3.68/24 = 0.153$$
$$< \mu_1(A_3) = 5.49/33 = 0.166$$
$$< \mu_1(A_4) = 7.242/40 = 0.181$$

$$< \mu_1(A_5) = 8.912/45 = 0.198$$
$$< \mu_1(A_6) = 10.412/48 = 0.216$$
$$< \mu_1(A_7) = 11.412/49 = 0.232.$$

Thus

$$\mu_1(A_1) < \mu_1(A_2) < \mu_1(A_3) < \mu_1(A_4) < \mu_1(A_5) < \mu_1(A_6) < \mu_1(A_7).$$

So we obtain the next hypergroup structure, according to Step 1:

$\circ_1$	A_1	A_2	A_3	$\cdots$	A_7
A_1	A_1	$\{A_1, A_2\}$	$\{A_1, A_2, A_3\}$	$\cdots$	H
A_2		A_2	$\{A_2, A_3\}$	$\cdots$	$\{A_2, A_3, \cdots, A_7\}$
A_3			A_3	$\cdots$	$\{A_3, \cdots, A_7\}$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
A_7				$\cdots$	A_7

Table 15

On the other hand, we get

$$\mu_2(A_1) = \mu_2(A_7) = 4.175/13 = 0.321$$
$$> \mu_2(A_2) = \mu_2(A_6) = 7.073/23 = 0.307$$
$$> \mu_2(A_3) = \mu_2(A_5) = 8.64/29 = 0.297$$
$$> \mu_2(A_4) = 9.14/31 = 0.294, \text{ i.e.,}$$

$$\mu_2(A_1) = \mu_2(A_7) > \mu_2(A_2) = \mu_2(A_6) > \mu_2(A_3) = \mu_2(A_5) > \mu_2(A_4),$$

Hence we obtain the following table:

$\circ_2$	A_1	A_2	A_3	A_4	A_5	$\cdots$	A_7
A_1	$\hat{A}_1$	$\hat{A}_1 \cup \hat{A}_2$	$\hat{A}_1 \cup \hat{A}_2 \cup \hat{A}_3$	$\hat{A}_1 \cup \cdots \cup \hat{A}_4$	$\hat{A}_1 \cup \hat{A}_2 \cup \hat{A}_3$	$\cdots$	$\hat{A}_1$
A_2		$\hat{A}_2$	$\hat{A}_2 \cup \hat{A}_3$	$\hat{A}_2 \cup \cdots \cup \hat{A}_4$	$\hat{A}_2 \cup \hat{A}_3$	$\cdots$	$\hat{A}_1 \cup \hat{A}_2$
A_3			$\hat{A}_3$	$\hat{A}_3 \cup \hat{A}_4$	$\hat{A}_3$	$\cdots$	$\hat{A}_1 \cup \hat{A}_2 \cup \hat{A}_3$
A_4				$\hat{A}_4$	$\hat{A}_3 \cup \hat{A}_4$	$\cdots$	$\hat{A}_1 \cup \hat{A}_2 \cup \hat{A}_3 \cup \hat{A}_4$
A_5							$\hat{A}_1 \cup \hat{A}_2 \cup \hat{A}_3$
A_6							$\hat{A}_1 \cup \hat{A}_2$
A_7							$\hat{A}_1$

Table 16

On the other hand, We have

$$\mu_3(A_4) = 3.7/13 = 0.284$$
$$> \mu_3(A_3) = \mu_3(A_5) = 8.03/32 = 0.250$$
$$> \mu_3(A_1) = \mu_3(A_7) = (5.32 + 4/7)/24 = 5.89/24 = 0.245$$
$$> \mu_3(A_2) = \mu_3(A_6) = 8.7/36 = 0.241, \text{ i.e.,}$$

$$\mu_3(A_4) > \mu_3(A_3) = \mu_3(A_5) > \mu_3(A_1) = \mu_3(A_7) > \mu_3(A_2) = \mu_3(A_6).$$

This means that

$\circ_3$	A_1	A_2	A_3	A_4	A_5	A_6	A_7
A_1	$\hat{A}_1$	$\hat{A}_1 \cup \hat{A}_2$	$\hat{A}_1 \cup \hat{A}_3$	$H - \hat{A}_2$	$\hat{A}_1 \cup \hat{A}_3$	$\hat{A}_1 \cup \hat{A}_2$	$\hat{A}_1$
A_2	$\hat{A}_1 \cup \hat{A}_2$	$\hat{A}_2$	$H - \{A_4\}$	H	$H - \{A_4\}$	$\hat{A}_2$	$\hat{A}_1 \cup \hat{A}_2$
A_3	$\hat{A}_1 \cup \hat{A}_3$	$H - \{A_4\}$	$\hat{A}_3$	$\hat{A}_3 \cup \hat{A}_4$	$\hat{A}_3$	$H - \{A_4\}$	$\hat{A}_1 \cup \hat{A}_3$
A_4	$H - \hat{A}_2$	H	$\hat{A}_3 \cup \hat{A}_4$	$\{A_4\}$	$\hat{A}_3 \cup \hat{A}_4$	H	$H - \hat{A}_2$
A_5	$\hat{A}_1 \cup \hat{A}_3$	$H - \{A_4\}$	$\hat{A}_3$	$\hat{A}_3 \cup \hat{A}_4$	$\hat{A}_3$	$H - \{A_4\}$	$\hat{A}_1 \cup \hat{A}_3$
A_6	$\hat{A}_1 \cup \hat{A}_2$	$\hat{A}_2$	$H - \{A_4\}$	H	$H - \{A_4\}$	$\hat{A}_2$	$\hat{A}_1 \cup \hat{A}_2$
A_7	$\hat{A}_1$	$\hat{A}_1 \cup \hat{A}_2$	$\hat{A}_1 \cup \hat{A}_3$	$H - \hat{A}_2$	$\hat{A}_1 \cup \hat{A}_3$	$\hat{A}_1 \cup \hat{A}_2$	$\hat{A}_1$

Table 17

Also, we get

$$\begin{aligned}\mu_4(A_4) &= 3.7/13 = 0.284\\ &> \mu_4(A_3) = \mu_4(A_5) = 8.03/32 = 0.250\\ &> \mu_4(A_2) = \mu_4(A_6) = \mu_3(A_1) = \mu_3(A_7)\\ &= (5.32 + 4/7)/24 = 5.89/24 = 0.245\\ &> \mu_4(A_1) = \mu_4(A_7) = \mu_3(A_2) = \mu_3(A_6) = 8.7/36 = 0.241, \text{ i.e.,}\end{aligned}$$

$$\mu_4(A_4) > \mu_4(A_3) = \mu_4(A_5) > \mu_4(A_2) = \mu_4(A_6) > \mu_4(A_1) = \mu_4(A_7).$$

We obtain $A_i \circ_4 A_j = A_i \circ_2 A_j$ for all indices i, j, which means that $(H, \circ_4) \cong (H, \circ_2)$, $(H, \circ_5) \cong (H, \circ_3)$ and so on. Therefore the fuzzy grade of $(H, \otimes)$ is 3.

8. Conclusions

Calculating the fuzzy degree of a hypergroup is a topic analyzed from various particular classes of hypergroups (See [1, 8, 9, 10, 11, 13]).

In this paper, we analyze a hypergroup that occurs in genetics, so we focused our study on certain particular cases, as follows:

i If $n = 6$ then the fuzzy degree of $(H, \otimes)$ is 3.

ii If $n = 8$ then the fuzzy degree of $(H, \otimes)$ is 4.

Notice that for $n = 2s$ and s is odd, then the classes $[A_i]$ have 4 elements for all $i \in \{1, ..., (s-1)/2\}$ and $[A_{(s+1)/2}]$ has 2 elements in the table of $\circ_3$, while if s is even all classes $[A_i]$ have 4 elements for all $1 \leq i \leq s$.

iii If $n = 5$ then the fuzzy degree of $(H, \otimes)$ is 3.

iv If $n = 7$ then the fuzzy degree of $(H, \otimes)$ is 3.

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