Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 25, No. 3, (June 2023) pp. 255–264
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr
https://doi.org/10.30948/afmi.2023.25.3.255

@FMI

Compactness in q-rung orthopair fuzzy topological spaces

A. Haydar Eş

Reprinted from the
Annals of Fuzzy Mathematics and Informatics
Vol. 25, No. 3, June 2023

Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 25, No. 3, (June 2023) pp. 255–264
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr
https://doi.org/10.30948/afmi.2023.25.3.255

Compactness in q-rung orthopair fuzzy topological spaces

A. Haydar Eş

Received 21 March 2023; Revised 12 April 2023; Accepted 6 May 2023

Abstract. Yager [1] introduced the fundamental concept of an orthopair fuzzy-set. Türkarslan et al. [2] proposed the idea of q-rung orthopair fuzzy topology on q-rung orthopair fuzzy sets. In this paper, the concept of q-rung orthopair fuzzy compactness, q-rung orthopair fuzzy almost compactness and q-rung orthopair fuzzy near compactness are introduced and studied. Further, q-rung orthopair fuzzy almost compactness of an q-rung orthopair fuzzy regular open or regular closed in q-rung orthopair fuzzy topology are introduced and characterized. Also, we investigate the behavior of q-rung orthopair fuzzy compactness under several types of q-rung orthopair fuzzy continuos.

2020 AMS Classification: 03E72, 08A72

Keywords: q-rung orthopair fuzzy set, q-rung orthopair fuzzy topological space, q-rung orthopair fuzzy compactness.

Corresponding Author: A. Haydar Eş (haydares@baskent.edu.tr)

1. Introduction

In the literature, there are several kinds of the concept of fuzzy set which are proposed after Zadeh [3] introduced the concept of fuzzy sets. Intuitionistic fuzzy sets, Pythagorean fuzzy sets and soft sets introduced by Atannasov [4], Yager [1] and Molodtsov [5], respectively. Moreover, many researchers [6, 7, 8, 9, 10, 11, 12, 13] have investigated topology of these sets. Zadeh [3] introduced the fundamental concept of a fuzzy set. Later Chang [4] has defined fuzzy topological spaces. Since Atanassov [6] introduced the notion of intuitionistic fuzzy sets and Çoker [7] developed the idea of intuitionistic fuzzy topological spaces and investigated various counterpart versions of traditional topological properties such as compactness and continuity. Eş and Çoker [14] introduced the investigate fuzzy almost compactness and fuzzy near compactness in intuitionistic fuzzy topological spaces. Yager and

Abbasov [15], and Yager [1] introduced the notion of Pythagorean fuzzy set, and Yager [1] introduced the generalized membership grading concept named as q-rung orthopair fuzzy set. Eş [16] gave the notion of connectedness for Pythagorean fuzzy topological space. Riaz et al. [17] studied the concept of q-rung orthopair fuzzy topological space by following the idea of Chang. A q-rung orthopair fuzzy set is a robust approach for fuzzy modelling, computational intelligence, and multicriteria decision-making problems.The main purpose of this paper is to extend the notions of fuzzy compactness, intuitionistic fuzzy compactness and Pythagorean fuzzy compactness to the notion of q-rung orthopair fuzzy topological space. Also, we investigate the behavior of q-rung orthopair fuzzy compactness under several types of q-rung orthopair fuzzy continuous.

2. Preliminaries

In this section, we give some basic preliminaries required for this paper.

Definition 2.1 ([4]). A *fuzzy set* A of given a set X is defined with a function $\mu_A : X \to I$ which is called the *membership function* of A.

Definition 2.2 ([4]). Let X be a non-empty fixed set and I the closed interval $[0, 1]$. An *intuitionistic fuzzy set* (briefly, IFS) A is an object of the following form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \,|x \in X\},$$

where the mappings $\mu_A : X \to I$ and $\nu_A : X \to I$ denote the *degree of membership* (namely) $\mu_A(x)$ and *degree of non-membership* (namely) $\nu_A(x)$ for each element $x \in X$ to the set A respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Definition 2.3 ([1, 15]). A *Pythagorean fuzzy subset* A of a non-empty set X is a pair $A = (\mu_A, \nu_A)$ of a membership function $\mu_A : X \to I$ and a non-membership function $\nu_A : X \to I$ with $\mu_A^2 + \nu_A^2 = r_A^2$ for each element $x \in X$, where $r_A : X \to I$ is a function which is called the *strength of commitment of point x*.

Definition 2.4 ([1]). A *q-rung orthopair fuzzy set* $\widetilde{A}$ (briefly, q-ROF set) of a non-empty set X is a pair $(\mu_{\widetilde{A}}, \nu_{\widetilde{A}})$ of a membership function $\mu_{\widetilde{A}} : X \to I$ and a non-membership function $\nu_{\widetilde{A}} : X \to I$ with $\mu^q_{\widetilde{A}}(x) + \nu^q_{\widetilde{A}}(x) = r^q_{\widetilde{A}}(x)$ for any $x \in X$ and for a real number $q \geq 1$, where $r_{\widetilde{A}} : X \to I$ is a function which is called the *strength of commitment at point x*.

Corollary 2.5 ([1]). (1) *Any IFS is a q-ROF set for all $q \geq 1$.*
(2) *An IFS is a Pythagorean fuzzy set.*
(3) *Any Pythagorean fuzzy subset is a q-ROF set for $q \geq 2$.*

Definition 2.6 ([1]). Let $\widetilde{A} = (\mu_{\widetilde{A}}, \nu_{\widetilde{A}})$ and $\widetilde{B} = (\mu_{\widetilde{B}}, \nu_{\widetilde{B}})$ be two q-ROF sets of a set X. Then
(i) the *complement* of $\widetilde{A}$, denoted by $\widetilde{A}^c$, is defined by

$$\widetilde{A}^c = (\nu_{\widetilde{A}}, \mu_{\widetilde{A}}),$$

(ii) the *intersection* of $\widetilde{A}$ and $\widetilde{B}$, denoted by $\widetilde{A} \cap \widetilde{B}$, is defined by

$$\widetilde{A} \cap \widetilde{B} = (min\{\mu_{\widetilde{A}}, \mu_{\widetilde{B}}\}, max\{\nu_{\widetilde{A}}, \nu_{\widetilde{B}}\}),$$

(iii) the *union* of $\widetilde{A}$ and $\widetilde{B}$, denoted by $\widetilde{A} \cup \widetilde{B}$, is defined by

$$\widetilde{A} \cup \widetilde{B} = (max\{\mu_{\widetilde{A}}, \mu_{\widetilde{B}}\}, min\{\nu_{\widetilde{A}}, \nu_{\widetilde{B}}\}),$$

(iv) we say $\widetilde{A}$ is a *subset* of $\widetilde{B}$ or $\widetilde{B}$ *contains* $\widetilde{A}$ and we write $\widetilde{A} \subset \widetilde{B}$ or $\widetilde{B} \supset \widetilde{A}$, if

$$\mu_{\widetilde{A}} \leq \mu_{\widetilde{B}} \text{ and } \nu_{\widetilde{A}} \geq \nu_{\widetilde{B}}.$$

Definition 2.7 ([2])**.** Let $X \neq \varnothing$ be a set and let τ be a family of q-ROF sets of X satisfying the following axioms:

(T_1) $\widetilde{1}_X,\ \widetilde{0}_X \in \tau$,

(T_2) $\widetilde{A}_1 \cap \widetilde{A}_2 \in \tau$ for any $\widetilde{A}_1,\ \widetilde{A}_2 \in \tau$,

(T_3) $\bigcup_{j \in J} \widetilde{A}_i \in \tau$ for any family of $\left\{\widetilde{A}_i\right\}_{i \in J} \subset \tau$,

where J is an index set, $\widetilde{1}_X$ and $\widetilde{0}_X$ are q-ROF sets which are defined by $(1, 0)$ and $(0, 1)$ respectively. Then τ is called a *q-rung orthopair fuzzy topology* on X and the pair (X, τ) is called a *q-rung orthopair fuzzy topological space* (briefly, q-ROFTS). Furthermore, each member of τ is called a *q-ROF open set* (briefly, q-ROFOS) in X. A q-ROF set $\widetilde{A}$ is called a *q-ROF closed set* (briefly, q-ROFCS) in X, if $\widetilde{A}^c \in \tau$.

Definition 2.8 ([2])**.** Let X and Y be two non-empty sets, let $f : X \to Y$ be a function and let $\widetilde{A}$ and $\widetilde{B}$ be q-ROF sets of X and Y respectively.

(i) The membership and the non-membership functions of the *image* $f(\widetilde{A})$ of $\widetilde{A}$ are defined by: for each $y \in Y$,

$$\mu_{f(\widetilde{A})}(y) = \begin{cases} sup_{z \in f^{-1}(y)} \mu_{\widetilde{A}}(z) & \text{if } f^{-1}(y) \neq \varnothing \\ \\ 0 & \text{otherwise} \end{cases}$$

and

$$\nu_{f(\widetilde{A})}(y) = \begin{cases} inf_{z \in f^{-1}(y)} \mu_{\widetilde{A}}(z) & \text{if } f^{-1}(y) \neq \varnothing, \\ \\ 1 & \text{otherwise} \end{cases}$$

respectively.

(ii) The membership and the non-membership functions of the *pre-image* $f^{-1}(\widetilde{B})$ of $\widetilde{B} \subset Y$ are defined by: for each $x \in X$,

$$\mu_{f^{-1}(\widetilde{B})}(x) = \mu_{\widetilde{B}}(f(x)),\ \nu_{f^{-1}(\widetilde{B})}(x) = \nu_{\widetilde{B}}(f(x))$$

respectively.

Proposition 2.9 ([2])**.** *Let X and Y be two non-empty sets and $f : X \to Y$ be a function and let $\widetilde{A}$ and $\widetilde{B}$ be q-ROF sets of X and Y respectively. Then*

(1) $f^{-1}[(\widetilde{B})^c] = [f^{-1}(\widetilde{B})]^c$,

(2) $[f(\widetilde{A})]^c \subset f[(\widetilde{A})^c]$,

(3) $f(f^{-1}(\widetilde{B})) \subset (\widetilde{B})$,

(4) $\widetilde{A} \subset f^{-1}(f(\widetilde{A}))$,

(5) $f^{-1}(\widetilde{B_1}) \subset f^{-1}(\widetilde{B_2})$, *whenever* $\widetilde{B_1} \subset \widetilde{B_2}$ *for any* $\widetilde{B_1},\ \widetilde{B_2}$ *are q-ROF sets of* Y,

(6) $f(\widetilde{A_1}) \subset f(\widetilde{A_2})$, *whenever* $\widetilde{A_1} \subset \widetilde{A_2}$ *for any* $\widetilde{A_1}$, $\widetilde{A_2}$ *are q-ROF sets of* X.

Definition 2.10 ([2])**.** Let $\widetilde{A}$ and $\widetilde{U}$ be two q-ROF sets in a q-ROFTS X. Then $\widetilde{U}$ is called a *neghborhood* of $\widetilde{A}$, there exists a q-ROFOS $\widetilde{F}$ in X such that $\widetilde{A} \subset \widetilde{F} \subset \widetilde{U}$.

Definition 2.11 ([2])**.** Let (X, τ_1) and (Y, τ_2) be two q-ROFTSs and let $f : X \to Y$ be a function. If for any q-ROF set $\widetilde{A}$ of X and for any neighbourhood $\widetilde{V}$ of $f(\widetilde{A})$ there exists a neighbourhood $\widetilde{U}$ of $\widetilde{A}$ such that $f(\widetilde{U}) \subset \widetilde{V}$, then f said to be *q-rung orthopair fuzzy continuous*.

Definition 2.12 ([7])**.** Let (X, τ) be a q-ROFTS and let $\widetilde{A}$ be a q-ROF set of X. Then the *q-ROF interior* $int(\widetilde{A})$ and the *q-ROF closure* $cl(\widetilde{A})$ are defined as:

(i) $int(\widetilde{A}) = \bigcup\{\widetilde{G}|\widetilde{G}$ is a q-ROFOS in X and $\widetilde{G} \subseteq \widetilde{A}\}$, i.e., $int(\widetilde{A})$ is the q-ROF union of q-ROF open sets contained in $\widetilde{A}$,

(ii) $cl(\widetilde{A}) = \bigcap\{\widetilde{K}|K$ is a q-ROFCS in X and $\widetilde{A} \subseteq K\}$, i.e., $cl(\widetilde{A})$ is the q-ROF intersection of q-ROF closed supersets of $\widetilde{A}$.

Proposition 2.13 ([17])**.** *Let* (X, τ_1) *and* (Y, τ_2) *two q-ROFTSs and* $f : X \to Y$ *be a function. Then the following statements are equivalent:*

(1) f *is a q-rung orthopair fuzzy continuous,*

(2) $f[cl(\widetilde{A})] \subseteq cl(f[\widetilde{A}])$ *for each q-ROF set* $\widetilde{A}$ *of* X,

(3) $cl(f^{-1}[\widetilde{K}]) \subseteq f^{-1}[cl(\widetilde{K})]$ *for each q-ROF set* $\widetilde{K}$ *of* Y,

(4) $f^{-1}[int(\widetilde{K})] \subseteq int(f^{-1}[K])$ *for each q-ROF set* $\widetilde{K}$ *of* Y.

Theorem 2.14 ([17])**.** *Let* (X, τ) *be a q-ROFTS and* $\widetilde{A}$ *be a q-ROF set of* X. *Then*

(1) $cl(\widetilde{A}^c) = (int(\widetilde{A}))^c$,

(2) $int(\widetilde{A}^c) = (cl(\widetilde{A}))^c$.

3. q-rung orthopair fuzzy compactness

Here, we generalize the concept of Pythgorean fuzzy compact topological space to the case of q-rung orthopair fuzzy compact topological space.

Definition 3.1. Let (X, τ) be a q-ROFTS, let $\mathbf{U} = \{\widetilde{U}_j\}_{j\in J}$ be a family of q-ROFOSs in X and let $\mathbf{U}^*$ be a finite subfamily of $\mathbf{U}$.

(i) $\mathbf{U}$ is called a *q-rung orthopair fuzzy open cover* of X, if $\bigcup_{j\in J} \widetilde{U}_j = \widetilde{1}_X$.

(ii) $\mathbf{U}^*$ is called a *finite open subcover* of $\mathbf{U}$, if $\mathbf{U}$ is a q-rung orthopair fuzzy open cover of X and $\mathbf{U}^*$ is a q-rung orthopair fuzzy open cover of X.

Definition 3.2. Let $\mathbf{V} = \{\widetilde{V}_j\}_{j\in J}$ be a family of q-ROFCSs in X. Then we say that $\mathbf{V}$ *has the finite intersection property*, if for each finite subfamily $\mathbf{V}^*$ of $\mathbf{V}$,

$$\bigcap_{j\in F} \widetilde{V}_j \neq \widetilde{0}_X,$$

where F is a finite subset of J.

Definition 3.3. A q-ROFTS (x, τ) is said to be *q-rung orthopair fuzzy compact*, if every q-rung orthopair fuzzy open cover of X has a finite subcover.

Example 3.4. Let $X = \{\alpha_1, \alpha_2\}$ and let $\{\widetilde{A}_i : i = 1, 2, ...\}$ are 3-rung orthopair fuzzy subsets of X such that $\mu_{\widetilde{A}_i}$ and $\nu_{\widetilde{A}_i}$ are corresponding membership and non-membership functions of $\widetilde{A}_i$ for each $i = 1, 2, 3, ..., n, ...$
If

$$\mu_{\widetilde{A}_i}(\alpha_1) = \frac{i}{i+1},\ \nu_{\widetilde{A}_i}(\alpha_1) = \frac{1}{i+1},$$

$$\mu_{\widetilde{A}_i}(\alpha_2) = \frac{i+1}{i+2},\ \nu_{\widetilde{A}_i}(\alpha_2) = \frac{1}{i+3},$$

then the family of 3-rung orthopair fuzzy sets $\tau = \left\{\widetilde{1}_X, \widetilde{0}_X\right\} \cup \left\{\widetilde{A}_i : i = 1, 2, ..., n, ...\right\}$ is a 3-rung orthopair fuzzy topological space on X. Since the 3-rung orthopair fuzzy open cover $\left\{\widetilde{A}_i : i = 1, 2, 3..., n, ...\right\}$ has no finite subcover, i.e., (X, τ) is not 3-rung orthopair fuzzy compact.

Theorem 3.5. *Let (X, τ_X), (Y, τ_Y) be two q-ROFTSs and let $f : X \to Y$ be a q-rung orthopair fuzzy continuous surjection. If (X, τ_X) is q-rung orthopair fuzzy compact, then so is (Y, τ_Y).*

Proof. Let $\{\widetilde{U}_j\}_{j\in J}$be a q-rung orthopair fuzzy open cover of Y. Then by the definition of q-rung orthopair fuzzy continuity, $\left\{f^{-1}(\widetilde{U}_j\right\}_{j\in J}$ is a q-rung orthopair fuzzy open cover of X. Since (X, τ_X) is q-rung orthopair fuzzy compact, there exists a finite subfamily $\left\{f^{-1}(\widetilde{U}_i) : i = 1, 2, .., n\right\}$ such that $\bigcup_{i=1}^{n} f^{-1}(\widetilde{U}_i) = \widetilde{1}_X$. Thus we have

$$f(\bigcup_{i=1}^{n} f^{-1}(\widetilde{U}_i)) = \bigcup_{i=1}^{n} f(f^{-1}(\widetilde{U}_i)) \subseteq \bigcup_{i=1}^{n} \widetilde{U}_i = \widetilde{1}_Y.$$

So (Y, τ_Y) is q-rung orthopair fuzzy compact. □

Definition 3.6. Let (X, τ_X) be a q-ROFTS. Then (X, τ_X) is said to be:

(i) *q-rung orthopair fuzzy almost compact*, if every q-rung orthopair fuzzy open cover of X has a finite subcollection whose closures cover X,

(ii) *q-rung orthopair fuzzy nearly compact*, if every q-rung orthopair fuzzy open cover of X has a finite subcollection such that the interiors of closures of q-rung orthopair fuzzy subsets in this subcollection cover X.

It is clear that in q-rung orthopair fuzzy topological spaces, we have the following implications:

q-ROF compactness → q-ROF nearly compactness
→ q-ROF almost compactness.

But the reverse implications do not hold.

Example 3.7. Let $X = \{\alpha_1, \alpha_2\}$ and let $\left\{\widetilde{G}_i : i = 1, 2, ..\right\}$ are 3-rung orthopair fuzzy subsets of X such that $\mu_{\widetilde{G}_i}$ and $\nu_{\widetilde{G}_i}$ are corresponding membership and non-membership functions of $\widetilde{G}_i$ for each $i = 1, 2, 3, ...$
If

$$\mu_{\widetilde{G}_i}(\alpha_1) = 1 - \frac{1}{i},\ \nu_{\widetilde{G}_i}(\alpha_1) = \frac{1}{1+i},$$

$$\mu_{\widetilde{G}_i}(\alpha_2) = 1 - \frac{1}{1+i},\ \nu_{\widetilde{G}_i}(\alpha_2) = \frac{1}{i+2},$$

then the family of 3-rung orthopair fuzzy sets $\tau_X = \{\widetilde{1}_X, \widetilde{0}_X\} \cup \{\widetilde{G}_i : i = 1, 2, ...\}$ is a 3-rung orthopair fuzzy topological space on X.
Since $cl(\widetilde{G}_i) = \widetilde{1}_X$ and $int(cl(\widetilde{G}_i)) = \widetilde{1}_X$, (X, τ_X) is 3-rung orthopair fuzzy nearly compact. But the 3-rung orthopair fuzzy open cover $\{\widetilde{G}_i : i = 1, 2, ...\}$ has no finite subcover, i.e., (X, τ_X) is not 3-rung orthopair fuzzy compact.

Theorem 3.8. *A q-ROFTS (X, τ_X) is q-rung orthopair fuzzy almost compact if and only if for every family $\{\widetilde{U}_j\}_{j\in J}$ of q-ROFOSs in X having the finite intersection property, we have $\bigcap_{j\in J} cl(\widetilde{U}_j) \neq \widetilde{0}_X$.*

Proof. Let $\mathbf{U} = \{\widetilde{U}_j\}_{j\in J}$ be any family of q-ROFOSs in X having the finite intersection property.Assume the $\bigcap_{j\in J} cl(\widetilde{U}_j) = \widetilde{0}_X$. Then we $\bigcup_{j\in J} int(\widetilde{U}_j^c) = \widetilde{1}_X$. Since X is q-rung orthopair fuzzy almost compact, there exists a finite subfamily $\{\widetilde{U}_j^c : j = 1, 2, ..., n\}$ such that $\bigcup_{j=1}^{n} cl(int(\widetilde{U}_j^c)) = \widetilde{1}_X$. Thus $\bigcup_{j=1}^{n} cl(cl(\widetilde{U}_j))^c = \widetilde{1}_X$. On the other hand, $\bigcap_{j=1}^{n} int(cl(\widetilde{U}_j)) = \widetilde{0}_X$. But from $\widetilde{U}_j \subseteq int(cl(\widetilde{U}_j))$, we see that $\bigcap_{j=1}^{n} \widetilde{U}_j = 0_X$, which is a contradiction with the finite intersection property of the family.

Conversely, let $\{\widetilde{K}_j\}_{j\in J}$ be a q-rung orthopair fuzzy open cover of X, and assume that that there exists no finite subfamily of $\{\widetilde{K}_j\}_{j\in J}$, whose closures is not a cover of X. Now the family $\{(cl(\widetilde{K}_j^c))^c : j \in J\}$, since $(cl(\widetilde{K}_j^c))^c = int(\widetilde{K}_j^c)$ consists of q-rung orthopair fuzzy open subsets in X and has the finite intersection property. Thus $\bigcap_{j\in F} cl(cl(\widetilde{K}_j^c))^c \neq \widetilde{0}_X$. Hence $\bigcup_{j\in F} int(cl(\widetilde{K}_j)) \neq \widetilde{1}_X$, which is a contradiction with $\bigcup_{j\in J} \widetilde{K}_j = 1_X$, since $\widetilde{K}_j \subseteq int(cl(\widetilde{K}_j))$ for each $j \in J$. □

Definition 3.9. Let (X, τ) be a q-ROFTS and let $\widetilde{A}$ be a q-ROF set in X. Then $\widetilde{A}$ is called:

(i) a *q-rung orthopair fuzzy regular open set* in X, if $\widetilde{A} = int(cl(\widetilde{A}))$,

(ii) a *q-rung orthopair fuzzy regular closed set* in X, if $\widetilde{B} = cl(int(\widetilde{B}))$.

Theorem 3.10. *In a q-ROFTS X, the following conditions are equivalent:*

(1) *X is q-rung orthopair fuzzy almost compact,*

(2) *for every family $\{\widetilde{K}_j\}_{j\in J}$ of q-rung fuzzy regular closed sets such that $\bigcap_{j\in J} \widetilde{K}_j = 0_X$, there exists a finite subfamily $\{\widetilde{K}_j : j \in F \subseteq J\}$ such that $\bigcap_{j\in F} int(\widetilde{K}_j) = 0_X$,*

(3) *$\bigcap_{j\in J} cl(\widetilde{K}_j) \neq 0_X$ for each family $\{\widetilde{K}_j\}_{j\in J}$ of q-rung orthopair fuzzy regular open set having the finite intersection property,*

(4) *Every q-rung orthopair fuzzy regular open cover of X contains a finite subfamily whose closures cover X.*

Proof. (1)⇒(2): Let $\mathbf{K} = \{\widetilde{K}_j\}_{j\in J}$ be a family of q-rung orthopair fuzzy regular closed sets in X with $\bigcap_{j\in J} \widetilde{K}_j = \widetilde{0}_X$. Then $\bigcup_{j\in I} \widetilde{K}_j^c = \widetilde{1}_X$. Since $\widetilde{K}_j^c = int(cl(\widetilde{K}_j^c))$, we have $\bigcup_{j\in J} int(cl(\widetilde{K}_j^c)) = \widetilde{1}_X$. From the q-rung orthopair fuzzy almost compactness, it follows that there exists a finite subfamily $\{\widetilde{K}_j : j \in F \subseteq J\}$ of K such that $\bigcup_{j\in F} cl(int(cl(K_j^c))) = \widetilde{1}_X$. Thus we have

$$\bigcup_{j\in F} cl(int(cl(\widetilde{K}_j^c))) = \bigcap_{j\in F} [cl(int(cl(\widetilde{K}_j^c))]^c = \bigcap_{j\in F} int(cl(int(\widetilde{K}_j))) = \bigcap_{j\in F} int(\widetilde{K}_j) = \widetilde{0}_X.$$

(2)⇒(3): Let $\{\widetilde{K}_j\}_{j\in J}$ be a family of q-rung orthopair fuzzy regular open sets having the finite intersection property, and assume that $\bigcap_{j\in J} cl(\widetilde{K}_j) = \widetilde{0}_X$. Since $\{cl(\widetilde{K}_j)\}_{j\in J}$ is a family of q-rung orthopair fuzzy regular closed sets in X, there exists a finite subfamily $\{cl(\widetilde{K}_j) : j \in F \subseteq J\}$ such that $\bigcap_{j\in F} int(cl(\widetilde{K}_j)) = \bigcap_{j\in F} \widetilde{K}_j = \widetilde{0}_X$, which is a contradiction.

(3)⇒(4): The proof is similar to (2)⇒(3).

(4)⇒(1): The proof is straightforward. □

Definition 3.11. Let (X, τ_X), (Y, τ_Y) be two q-ROFTSs and let $f : X \to Y$ be a function. Then f is called:

(i) *q-rung orthopair fuzzy almost continuous*, if the pre-image of each q-rung orthopair fuzzy regular open set in Y is a q-rung orthopair fuzzy open set in X,

(b) *q-rung orthopair fuzzy weakly continuous*, if for each q-rung orthopair fuzzy open set $\widetilde{U}$ in Y, $f^{-1}(\widetilde{U}) \subseteq int(f^{-1}(cl(\widetilde{U})))$.

Theorem 3.12. *Let (X, τ_X), (Y, τ_Y) be two q-ROFTSs and let $f : X \to Y$ be a q-rung fuzzy almost continuous surjection. If X is q-rung orthopair fuzzy almost compact, then so is Y.*

Proof. Let $\{\widetilde{U}_j\}_{j\in J}$ be a q-rung orthopair fuzzy open cover of Y. Since f is q-rung orthopair fuzzy almost continuous, $\{f^{-1}(int(cl(\widetilde{U}_j)))\}_{j\in J}$ is a q-rung orthopair fuzzy open cover of X. Since X is q-rung orthopair fuzzy almost compact, there exist $\{\widetilde{U}_j : j \in F \subseteq J\}$ such that $\bigcup_{j\in F} cl(f^{-1}(int(cl(\widetilde{U}_j)))) = \widetilde{1}_X$. From the surjectivity and q-rung orthopair fuzzy almost continuity of f, we obtain

$$f(\bigcup_{j\in F} cl(f^{-1}(int(cl(\widetilde{U}_j))))) = \bigcup_{j\in F} cl(f^{-1}(int(cl(\widetilde{U}_j)))) = f(\widetilde{1}_X) = \widetilde{1}_Y.$$

Since $f(int(f^{-1}(cl(\widetilde{U}_j)))) \subseteq f(f^{-1}(cl(\widetilde{U}_j)) = cl(\widetilde{U}_j)$, we deduce

$$f(int(f^{-1}(cl(\widetilde{U}_j)))) \subseteq cl(\widetilde{U}_j) \text{ for all } j \in J.$$

Thus $\bigcup_{j\in F} cl(\widetilde{U}_j) = \widetilde{1}_Y$. So Y is also q-rung orthopair fuzzy almost compact. □

Theorem 3.13. *Let (X, τ_X), (Y, τ_Y) be two q-ROFTSs and let $f : X \to Y$ be a q-rung orthopair fuzzy weakly continuos fuzzy surjection. If X is q-rung orthopair fuzzy compact, then so Y.*

Proof. The proof is similar to Theorem 3.12. □

Definition 3.14. Let (X, τ_X), (Y, τ_Y) be two q-ROFTSs and let $f : X \to Y$ be a function. Then f is said to be *q-rung orthopair fuzzy strongly continuous*, if for each q-ROF set $\widetilde{G}$ in X, $f(cl(\widetilde{G})) \subseteq f(\widetilde{G})$.

Theorem 3.15. *Let (X, τ_X), (Y, τ_Y) be two q-ROFTSs and let $f : X \to Y$ a q-rung orthopair fuzzy strongly continuous surjection. If X is q-rung orthopair fuzzy almost compact, then Y is q-rung orthopair fuzzy compact.*

Proof. The proof follows a similar pattern to Theorem 3.12. □

Definition 3.16. Let (X, τ_X), (Y, τ_Y) be two q-ROFTSs and let $f : X \to Y$ be a function. Then f is said to be *q-rung orthopair fuzzy almost open*, if the image of each q-rung othopair fuzzy regular open set in X is a q-ROFOS in Y.

Theorem 3.17. *The image of a q-rung orthopair nearly compact fuzzy topological space under a q-rung orthopair fuzzy almost continuous and almost open function is q-rung orthopair fuzzy nearly compact.*

Proof. The proof follows a similar pattern to Theorem 3.12. □

Definition 3.18. A q-ROFTS X is said to be *q-rung orthopair fuzzy countably compact*, if every countable q-rung orthopair fuzzy open cover of X has a finite subcover.

Example 3.19. Let $X = \{\alpha_1, \alpha_2\}$ and define the 3-rung orthopair fuzzy subsets $\{\widetilde{A}_i : i \in \mathbb{N}\}$ as follows:

$$\mu_{\widetilde{A}_i}(\alpha_1) = \frac{i}{i+1}, \ \nu_{\widetilde{A}_i}(\alpha_1) = \frac{1}{i+1} \text{ and}$$
$$\mu_{\widetilde{A}_i}(\alpha_2) = \frac{1}{i+2}, \ \nu_{\widetilde{A}_i}(\alpha_2) = \frac{1}{i+3} \ .$$

Then the family $\tau_X = \{\widetilde{0}_X, \widetilde{1}_X\} \cup \{A_i : i \in \mathbb{N}\}$ is an 3-rung orthopair fuzzy topological space on X. The 3-rung orthopair fuzzy topological space (X, τ_X) is not 3-rung orthopair fuzzy countably compact.

Example 3.20. Let $X = [0, 1]$. Consider the 3-rung orthopair fuzzy sets $\widetilde{A}_i$, for $i = 2, 3, 4, \cdots$, and $\widetilde{A}$ in X defined as follows:

$$\mu_{\widetilde{A}_i}(x) = \begin{cases} \frac{2}{5} & \text{if } x = 0 \\ ix & \text{if } 0 < x \leq \frac{1}{i} \\ 1 & \text{if } \frac{1}{i} < x \leq 1, \end{cases} \qquad \nu_{\widetilde{A}_i}(x) = \begin{cases} \frac{1}{10} & \text{if } x = 0, \\ 1 - ix & \text{if } 0 < x \leq \frac{1}{i} \\ 0 & \text{if } \frac{1}{i} < x \leq 1 \end{cases}$$

and

$$\mu_{\widetilde{A}}(x) = \begin{cases} \frac{2}{5} & \text{if } x = 0 \\ 1 & \text{otherwise}, \end{cases} \qquad \nu_{\widetilde{A}}(x) = \begin{cases} \frac{1}{10} & \text{if } x = 0 \\ 0 & \text{otherwise}. \end{cases}$$

Then $\tau_X = \{\widetilde{0}_X, \widetilde{1}_X, A\} \cup \{A_i : i = 2, 3, 4, ...\}$ is a 3-rung orthopair fuzzy topological space on X. The 3-rung orthopair fuzzzy topological space (X, τ_X) is 3-rung orthopair fuzzy countably compact, since every countable open cover of X should contain $\tilde{1}_X$.

4. Conclusion

This paper introduces certain properties of q-rung orthopair fuzzy compactness are significant results of q-rung orthopair fuzzy compactness. We give some characterizations of q-rung orthopair fuzzy compactness in terms of q-rung orthopair fuzzy regular open sets or q-rung orthopair fuzzy regular closed sets. Moreover, we introduce and study q-rung orthopair fuzzy near compactness. We examine multiple relationships between different types of q-rung orthopair fuzzy compactness. The results in this work can be extended to the q-rung orthopair fuzzy seperation axioms.

5. Acknowledgement

The author would like to thank the referees for their valuable comments and suggestions.

References

[1] R. R. Yager, Generalized orthopair fuzzy sets, IEEE Transactions on fuzzy systems 25 (2017) 1222–1230.
[2] E. Türkaslan, M. Ünver and M. Olgun, q-rung orthopair fuzzy topological spaces, Lobachevskii Journal of Mathematics 42 (2021) 470–478.
[3] L. A. Zadeh, Fuzzy Sets, Information and Control 8 (1965) 338–353.
[4] K. T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986) 87–96.
[5] D. Molodtsov, Soft sets-first results, Comput. Math. Appl. 37 (1999) 19–31.
[6] C. L. Chang, Fuzzy topological spaces, J. Math. Anal. Appl. 24 (1968) 182–190.
[7] D. Çoker, An introduction to intuitionistic fuzzy topological spaces, Fuzzy Sets and Systems 88 (1997) 81–89.
[8] G. Şenel, Soft topology generated by *L*-soft sets, Journal of New Theory 4 (2018) 88–100.
[9] G. Şenel, A new approach to Hausdorff space theory via the soft sets, Mathematical Problems in Engineering 9 (2016) 1–6.
[10] G. Şenel and N. Çağman, Soft topological subspaces, Ann. fuzzy Math. Inform. 10 (4) (2015) 525–535.
[11] G. Şenel and N. Çağman, Soft closed sets on soft bitopological space, Journal of New results in Science 3 (2014) 57–66.
[12] G. Şenel, J. G. Lee and K. Hur, Distance and similarity measures for octahedron sets and their application to MCGDM Problems, Mathematics 8 (2020) 1690.
[13] J. G. Lee, G. Şenel, P. K. Lim, J. Kim and K. Hur, Octahedron sets, Ann. Fuzzy Math. Inform. 19 (3) (2020) 211–238.
[14] D. Çoker and A. H. Eş, On fuzzy compactness in intuitionistic fuzzy topological spaces, Journal of Fuzzy Mathematics 3 (1995) 899–910.
[15] R. R. Yager and A. M. Abbasov, Pythagorean membership grades, complex numbers and decision making, International Journal of Intelligent Systems 28 (2013) 436–452.
[16] A. H. Eş, Connectedness in Pythagorean fuzzy topological spaces, International Journal of Mathematics Trends and Technology 65 (2019) 110–116.
[17] M. Riaz, H. M. A. Farid, S. A. Alblowi and Y. Almalki, Novel concepts of q-rung orthopair fuzzy topology and WPM approach for multicriteria decision-making, Journal of Function Spaces 5 (2022) 1–16 Article ID: 2094593.

A.Haydar Eş (haydares@baskent.edu.tr) – Department of mathematics education, faculty of education, Başkent University, postal code 06790, Ankara, Turkey.